The Mathematics Department offers a doctorate in Mathematics, an interdisciplinary doctorate in Mathematics/Scientific Computation, and a doctorate in Applied and Interdisciplinary Mathematics (AIM).

The requirements for the AIM program are substantially different from the other two and are listed separately. Most of this document describes in detail requirements for the doctorate in Mathematics. The final section describes the requirements for the doctorate in Mathematics/Scientific Computation.

I. Admission To The Doctoral Program

Every winter, the Admissions and Fellowships Committee of the Department of Mathematics reviews applications to the graduate programs and admits students for the following Fall term.

II. Overview Of The Program

The goal of the Doctoral Program is to produce graduates well-prepared to go on to successful careers in the mathematical sciences, whether in academic, governmental, or industrial positions. While the thesis is the centerpiece of the program, for many careers the breadth provided by the various course requirements is of equal importance.

Each student must pass, or demonstrate a knowledge of, six of the eight core courses. These consist of two-course sequences in algebra (593, 594), analysis (596, 597), applied analysis (556, 572), and geometry/topology (591, 592).

Each student must pass the Qualifying Review. This consists of written examinations, based on the same syllabuses as the core courses, a course requirement, and a survey by the Doctoral Committee of the student's record as a graduate student in the Department. The purpose of the Review is to ensure that students have a good knowledge of core graduate mathematics and to evaluate the chances that a student will be able to complete a Ph.D. degree. The Qualifying Review Examination is conducted three times a year, and should be taken as soon as the student feels ready. It can be taken as many times as necessary with the only stipulation that a student must pass an exam in one area by the beginning of the fourth term in the program, and must complete the entire Review by the beginning of the sixth.

Students are required to take six courses beyond those needed for the Qualifying Review, distributed among at least three of five areas of mathematics.

To ensure greater intellectual breadth, the Graduate School requires that every student must successfully complete four hours of cognate courses before achieving Candidacy. For students in most departments, these must be taken outside the student's home department, but mathematics students are allowed to take courses within the Mathematics Department under certain restrictions subject to the approval of the Doctoral Committee. Cognate courses can be taken at any time.

The course requirements listed above should be regarded as the absolute minimum. The Department expects that most students will take more courses distributed so that they achieve a broad background in their specialty and related areas. Students should also participate actively in the Departmental Seminars offered in their area of interest and attend Colloquia since it is there that they can learn about the latest developments and open problems.

To ensure that, during their careers, students have access to mathematics written in languages other than English, all students are required to demonstrate a reading knowledge of mathematics in one of the remaining major scientific languages; French, German, and Russian.

Students entering the program are initially counseled by members of the Doctoral Committee. They are encouraged to begin thinking early about what area of mathematics they will write their theses in, and also about possible thesis Advisors. All faculty members are available to assist the student with this process. Once a student has decided on an area, he or she should seek advice on an informal basis from the faculty members in that area, especially from potential thesis Advisors. Students who already have an area of special interest, should consult with faculty in that area on their arrival.

After passing the Qualifying Review, students consult with the Chair of the Doctoral Committee about the choice of an Advisor. Ideally, the Advisor is a faculty member who is in the area of greatest mathematical interest to the student, who is known to the student perhaps through taking a course or seminar, and with whom the student will be able to write a thesis. Although the Advisor can be changed, it is important that he or she be chosen with great care. To assist students in their choice of an Advisor, students may view research areas on faculty home pages on the departmental web site.

After the Qualifying Review, the next formal step in the program is admission to Candidacy for which a student must have completed the language examination, the cognate courses, and the Preliminary Examination, and should have completed the six-course distribution requirement.

The purpose of the Preliminary Examination is to ensure that students have the necessary background to complete a thesis in their chosen area and to test whether the student has the ability to make the transition from course work to thesis work. The student is examined on a list of topics drawn up in consultation with the student's Advisor and approved by the Doctoral Committee. The content of the list of topics is roughly that of two advanced courses, but should include material that the student has learned individually, by reading books or papers.
After achieving Candidacy, students must take one graduate mathematics course for credit during each of the next two terms in residence, but they are encouraged to take a course every term.

The final stage in the program is the completion of the research and the writing of the thesis. This is certainly the most difficult and least predictable part of the program since the student is required to make an original contribution to mathematics. The chances for success are enhanced by selecting an area to work in and consulting with an appropriate faculty Advisor as early as possible. Normally a good student who comes into the program adequately prepared will find a thesis Advisor and start to read and even do research not long after passing the Qualifying Review Examination, certainly by the time he or she achieves Candidacy.

To remain in the Ph.D. program, students must progress at a rate deemed satisfactory by the Doctoral Committee. The Department cannot support a graduate student for more than ten terms with College funds. Since the research and writing of a thesis usually take at least two years, students should achieve Candidacy by the end of their third year at the latest. In order to meet this requirement, students will normally need to choose an Advisor by the start of their sixth term. Students will find that enrollment in the doctoral program requires full concentrated effort, and is much more intensive than an undergraduate program.

Students should also be aware of the Graduate School requirements (see the Rackham Student Handbook). For example, a student is required to maintain at least a B average in courses, and must complete all doctoral work within seven years of the same date (students may petition for extra time). A student's fees drop substantially when Candidacy is achieved, but a Candidate may elect only one course per full term without payment of additional fees.

III. Stage 1

This period begins with admission to the Doctoral Program and ends with completion of the Qualifying Review. Students must complete this part of the Program by the beginning of their sixth term, but the expectation is that it will be completed much earlier.

Students at Stage 1 are assigned a member of the Doctoral Committee as their counselor. They may also seek advice from any faculty member. However, only members of the Doctoral Committee can approve their course elections or changes.

Students meet with their counselors before each term to decide which courses they will take. Normally students should take two core courses in each of their first two terms, together with one or two other courses, which may be outside the core areas, for example, in the student's area of special interest.

The Qualifying Review Examination. This is a written examination, offered three times a year in early January, May, and September, which covers applied analysis and each of the three major areas of pure mathematics—algebra, analysis, and geometry/topology. The syllabus for the Examination in each area is the same as that for the corresponding core courses:

- **Algebra:** Math 593, 594
- **Analysis:** Math 596, 597
- **Applied Analysis:** Math 556, 572
- **Geometry/Topology:** Math 591, 592

Students are responsible for the material in the syllabus even when it is not covered in a particular core course. Detailed syllabuses and copies of old exams are available in the Graduate Student Office.

**The Qualifying Review.** To pass the Qualifying Review a student must pass the Qualifying Review Examination in two of the four areas, and pass three further courses satisfying the following requirements:

- Students who pass the Qualifying Review Exams in two of the pure areas, algebra, analysis, or geometry/topology, must pass the two core courses in the third pure area, or one core course in each of the remaining two areas, with at least a B average and no grades C+ or lower. For example, a B+ in one course and a B- in the other qualifies; an A+ in one and a C+ in the other does not. Such students must also successfully complete one further course in an area outside the pure core areas, for example, in algebraic geometry, applied mathematics, combinatorics, differential geometry, logic, mathematical physics, number theory, numerical analysis, or probability. An applied core course may be used for this purpose.

- Students who pass the Qualifying Review Exam in Applied Analysis and one other area must pass one core course in each of the remaining two areas, with at least a B average and no grades of C+ or lower. Such students must also successfully complete one additional course in algebra, pure analysis, geometry, topology, or discrete mathematics.

In each case, a student may pass the Qualifying Review Examination in an area instead of passing the two core courses in the area. With the permission of the Chair of the Doctoral Committee, a student may substitute a more advanced course in the same immediate area for a core course, for example, the student may substitute a more advanced complex analysis course for the complex analysis core course, but not for the real analysis core course.

When the Qualifying Review Examination has been graded, the Doctoral Committee decides on the basis of the Examination results and the entire academic record whether a student should pass the Review. Additional faculty input is often sought before making a final decision. For a student who has passed two examinations but has yet to complete the course work, the Committee may pass the student subject to successful completion of the courses.

As indicated above, students are encouraged to take the Examination as soon as possible. One of the two written exams must be passed by the start of the fourth term. The Department expects the entire Qualifying Review to be completed by the end of the fourth term, and requires that it be completed by the start of the sixth term. Thus, for example, a student entering the Doctoral Program in the Fall of 2003 must pass the Examination in at least one area by early January 2005 and the whole Review by no later than early January 2006. Normally, the Fellowship Committee will not commit to financial aid beyond the second year until the Qualifying Review has been passed.
IV. Stage 2

This period commences with successful completion of the Qualifying Review and finishes with admission to Candidacy. Most students complete this phase in 12 to 18 months.

Advisor. After passing the Qualifying Review Examination, students consult with the Chair of the Doctoral Committee about the choice of a Advisor. Ideally, the Advisor will supervise the student through the completion of the student's degree, but the student may make a change of Advisor, after consulting with the Chair of the Doctoral Committee, if, for example, the student's field of interest changes.

Long Range Plan of Study. Each student entering Stage 2 should prepare a long-range study plan in consultation with his or her Advisor. This means making tentative choices about the topics for the Preliminary Examination (including a detailed list of subjects) as well as for a foreign language and cognates.

Requirements for Candidacy. Admission to Candidacy is contingent upon completing the following four requirements:

1. Knowledge of six mathematics courses chosen from at least three of five areas.
2. Passing the Preliminary Examination.
3. Passing the Departmental Reading Examination in one of the following three languages—French, German, Russian.
4. Completion of four hours of cognate courses.

The course requirement. Each student must earn a grade of at least B- in six advanced mathematics courses, in addition to core courses or substitutes used to fulfill the requirements for the Qualifying Review, and chosen from three of the following five areas:

a) algebra, algebraic geometry, algebraic number theory;

b) analysis, analytic number theory, probability;

c) topology, differential geometry;

d) applied analysis, numerical analysis;

e) applied discrete mathematics, combinatorics, logic.

Eligible courses include those at the 600 level or above and the more advanced 500 level courses. Transfer students may petition the Doctoral Committee to receive credit for upper level courses taken at another institution. With the approval of the Doctoral Committee, certain courses taken outside the department, for example, in physics may be allowed to count for this requirement (under d).

The Department may allow the student to proceed to Candidacy before fulfilling this course distribution requirement. If the Chair of the Doctoral Committee determines that the student has not fulfilled the requirement at the time the student applies for Candidacy, he or she will notify the student, who must then meet with him to agree on a program to complete the requirement.

Preliminary Examination. This examination is normally in the area of the student's thesis. The material covered should be equivalent to at least two 600 or 700 level courses, but should include material the student has learned individually.

In order to pass the Preliminary Examination the student must pass an oral examination. The Preliminary Examination is administered by a Preliminary Examination Committee and is based on a detailed list of topics prepared by the student in consultation with the Advisor. The Advisor usually serves as Chair of the Committee. The Advisor also advises the student on the choice of an additional faculty member of the Committee. The list of subjects for the oral examination must be approved and initialed by all members of the Preliminary Examination Committee and by the Chair of the Doctoral Committee before the examination can take place.

When a date for the Examination has been agreed on by the student and examiners, it should be reported to the Graduate Student Office, which will arrange for a room and officially remind each examiner as the date approaches.

Students who fail the Preliminary Examination are granted at most one opportunity to take it again. This exam must be passed by the beginning of the student's eighth term in the program.

Language Requirement. Students in the Doctoral Program are required to pass Departmental Reading Examinations in one of the following languages:

French, German, Russian.

These examinations are given twice a year, in mid-October and mid-February, by members of the faculty. The examiners may be willing to give exams at other times if students have a special need. Students are strongly advised to satisfy this requirement as early in their program as possible.

The Department recommends that students develop some sort of proficiency in all three languages.

Cognate Requirements. The Graduate School requires that every student must successfully complete four hours of cognate courses before achieving Candidacy.

These may be either

a) graduate courses offered by a Department other than the Mathematics Department, or

b) courses at the 500 level or above offered by the Mathematics Department for which a grade of B or better is earned, provided these courses

(i) treat ideas, techniques, or patterns of problem solving distinctly different from those of the student's major area;

(ii) involve significant intellectual content that is important in an area of science other than Mathematics;

(iii) are approved in advance by the Doctoral Committee of the Mathematics Department.

For courses taken within the Mathematics Department, a written statement must be provided by the student at the time approval is sought, cosigned by the student's Advisor, explaining how the specific course meets conditions (i) and (ii). In certain cases the Doctoral Committee may be willing to approve a mathematics course as a cognate after it has been taken provided the course clearly satisfies the conditions (i) and (ii).

A document giving advice on the selection of cognate courses is available from the Graduate Student Office.

Candidacy. When a student has completed all of the requirements listed above---six courses distributed over three of five areas, the Preliminary Examination, the Foreign

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Eligible courses are those listed as graduate courses in the Rackham “Program Information and Course Offerings”. It includes most, but not all, courses at the 400 level and above.
Language Requirement, the Cognate Requirement—the Department recommends to the Graduate School that the student be admitted to Candidacy.

V. Stage 3

This period begins with admission to Candidacy and finishes with a successful Thesis Defense. This is the most difficult part of the Program. No student should expect to complete it in less than twenty-four months.

Post Candidacy Course Requirement. After entering Candidacy a student is required to take one graduate mathematics course for credit during each of the next two semesters in residence.

Thesis Advisor. A student should normally be involved in thesis research by the time he or she achieves Candidacy, and the Advisor becomes the student's Thesis Advisor.

Thesis Research. Students normally do thesis research in the major area of the Preliminary Examination.

Most students find that doing research is very different from attending courses and participating in seminars. In research, one is alone for the first time, trying to develop mathematical ideas from unfamiliar material without the security of knowing that the answers have been worked out in advance. Luck, talent, inspiration and perseverance certainly contribute to success. Moreover, the last quality in sufficient quantity can help overcome deficiencies in the first three. Involvement with one's work is, without a doubt, an essential requirement.

The time required to complete this phase of the Program should not be underestimated.

Dissertation. When the Thesis Advisor is satisfied that a student has done sufficient research for the Ph.D., the student prepares a draft of the dissertation. The Doctoral Committee then appoints two Readers to submit independent evaluations and decides on the basis of these reports whether or not to approve the work.

The student's Thesis Advisor usually serves as one Reader. The student must give the Readers sufficient time to read the draft so that they can react and pass on valuable suggestions. In any case, the draft must be in the hands of the Readers at least eight weeks before their evaluation is required.

After the draft has been approved by the Doctoral Committee, the student prepares a final manuscript in accordance with the many detailed regulations described in the Graduate School's "Dissertation Handbook", available in the Rackham Building. The student should obtain a copy of this handbook early in the term in which the thesis is expected to be completed, because many of Rackham's deadlines, for example that for a format check, are strictly enforced. The thesis should not be printed in final form prior to the approval of the Doctoral Committee.

Dissertation Committee. The Dissertation Committee consists of five members, four of whom should be from Mathematics and one of whom must be from outside the Department. It is usually chaired by the Thesis Advisor and normally includes both Readers of the student's thesis. One of the members from the Mathematics Department must be from outside the student's area. The student is expected to ask faculty members to be on the Committee after consultation with the Thesis Advisor and approval by the Doctoral Committee concerning its composition. The Chair of the Doctoral Committee can sometimes help in suggesting members for the Committee from outside the Department. The student would be well advised to choose the Committee early so they may be consulted as the research progresses.

Thesis Defense. When the thesis is in final form, the student is examined orally on its contents and related topics by the Dissertation Committee. Copies of the Thesis should be in their hands at least two weeks before the Thesis Defense. The Graduate School should be notified by the Graduate Student Office of the time and place for the examination at least one week before it is to take place. It is advisable to make these arrangements as far in advance as possible as a matter of courtesy and to avoid dates when members of the Committee may be out of town; this is especially so if the exam is to occur during the summer months. There are several other important deadlines which must be met before a Dissertation Defense can be scheduled—see the "Dissertation Handbook".

VI. Scientific Computation Program

The Mathematics Department offers an interdisciplinary doctorate in Scientific Computation in cooperation with the Laboratory for Scientific Computation (LaSC) and several Departments in the College of Engineering. Students seeking the Mathematics/Scientific Computation degree are students in the Mathematics Department and proceed to the degree in the same three stages described above, but they should also make contact with LaSC as soon as it is decided that they will seek this degree. LaSC will appoint a committee to counsel and supervise the student.

Stage 1

To pass the Qualifying Review, students in the Mathematics/Scientific Computation degree program must:

1) Pass the Qualifying Review in one of the three core areas of Algebra, Analysis and Geometry/Topology.

2) Pass a written exam in numerical analysis to be given by the faculty of LaSC. The material for this exam is covered in Math 571 and 572.

3) Either pass the Qualifying Review in a second of the three core areas of Algebra, Analysis and Geometry/Topology, or pass two additional core courses, each with a grade of B or higher, chosen from two of the four core areas of Algebra, Analysis, Applied Analysis and Geometry/Topology, but excluding Math 572 which has already been used in 2).

As is the case for other Mathematics students one of the two written exams must be passed by the start of the fourth term, it is expected that the entire Review will be completed by the end of the fourth term and must be completed by the start of the sixth term. Normally financial aid is not committed beyond the second year until the Qualifying Review has been passed.

Stage 2

At this stage the student should have a LaSC Advisory Committee and should in reasonable time select a member of the Mathematics Department faculty, also a member of LaSC, to serve as his or her supervisor and chair of the Advisory Committee.
Requirements for Candidacy in Mathematics/Scientific Computation

Admission to Candidacy in this program is contingent upon completing the following requirements:

1. Knowledge of at least five Mathematics courses at the level 500 or above in addition to those used to satisfy the Qualifying Review, and including at least one additional course in numerical analysis, chosen so as to meet the criterion in "The course requirement" on page 4. One of the five could be a core course not used to meet the Qualifying Review requirements.

2. Knowledge of at least three graduate courses in scientific computation or applications offered by associated Departments. (This requirement also fulfills the Cognate requirement).

3. Passing the Departmental Reading Exam in one of the following three languages: French, German, Russian.

4. Passing the oral Preliminary Exam, which will be administered by two mathematicians and two additional faculty from LaSC selected with the approval of both LaSC and the Chair of the Doctoral Committee. The student will be examined on a detailed list of subjects in scientific computation and related areas prepared by the student in consultation with the Advisor.

Stage 3

A. Post Candidate Course Requirement

After entering Candidacy a student is required to take one advanced course in mathematics, scientific computation, or applications for credit during each of the next two semesters in residence.

B. Thesis Advisor/Thesis Topic

The Thesis Advisor is chosen by the student with the approval of LaSC and the Doctoral Committee of the Mathematics Department. The thesis will be on some aspect of scientific computation and will be directed by the Advisor and a committee appointed by LaSC and the Doctoral Committee which will usually serve as the Dissertation Exam Committee.

The steps in preparation of the Dissertation and its defense are the same as on page 5.