Department of Mathematics
Winter 2013 Graduate Course Descriptions

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501 AIM Student Seminar Miller F 11am-12:00pm & 3-4pm

Prerequisites: *You must be a graduate student in the AIM program to register for this course.*

"Math 501 is a required course for all students enrolled in the Applied and Interdisciplinary Mathematics (AIM) MS and PhD graduate programs. In the Winter term, all first-year AIM students from both programs must sign up for this course. Due to the highly specialized content of the course, enrollment is available only for students in an AIM degree program.

The purpose of Math 501 is to address specific issues related to the process of studying applied mathematics in the AIM program and becoming an active member of the research community. The weekly meetings of the class will be divided among three types of sessions:

1. "Focus on. . . " presentations. These are presentations on various topics, some of immediate practical significance for students and others of a further-reaching nature. These discussions will include aspects of scholarly writing, research, and career development.

2. AIM Faculty Portraits. These are short presentations by faculty members in the Mathematics Department and other partner disciplines who are potential advisors or committee members for AIM students. The AIM faculty portraits provide a direct channel for students to discover what research is being done in various areas by current faculty, and to see what kind of preparation is required for participating in such research.

3. AIM Research Seminar Warm-up talks. One of the course requirements for Math 501 is weekly attendance of the AIM Research Seminar that takes place from 3-4 PM each Friday. The warm-up talks are presentations during the regular course meeting time by particularly dynamic speakers slated to speak in the AIM Research Seminar later the same day as a way to provide background material with the goal of making the AIM Research Seminar lecture more valuable for students.

Weekly attendance both of the course meeting and also of the AIM Research Seminar is required for Math 501. If you are registering for Math 501 you must be available both during the regular class time of 11am-12pm on Fridays as well as during the AIM Research Seminar which runs 3-4pm on Fridays. If you are teaching, you should keep both of these obligations in mind when you submit your class/seminar schedule prior to obtaining a teaching assignment. Other requirements, including possible assignments related to topics discussed in the lectures, will be announced by the instructor in class."

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521 Life Contingencies II Moore TTh 10:00 - 11:30/ TTh 11:30-1:00

Prerequisites: *Math 520 or permission*

Frequency: Winter (II)

Background and Goals: This course extends the single decrement and single life ideas of Math 520 to multi-decrement and multiple-life applications directly related to life insurance. The sequence 520--521 covers covers the material for Examination 3L of the Casualty Actuarial Society and for Examination MFE of the Society of Actuaries.

Content: Topics include multiple life models--joint life, last survivor, contingent insurance; multiple decrement models---disability, withdrawal, retirement, etc.; and reserving models for life insurance. This corresponds to chapters 7--11 and 15 of Bowers et al.

Subsequent Courses: none

Text: Actuarial Mathematics 2, Bowers, Gerber, Hickman, Jones, Nesbitt 0-938959-46-8 Required

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523 Risk Theory Marker TTh 11:30-1:00/ TTh 1:00-2:30

Prerequisites: *Math 425*

Course Description: The course studies actuarial models, focusing on topics covered by professional examinations C (Society of Actuaries) and 4 (Casualty Actuarial Society).. It covers the broad areas of (1) actuarial distributions, (2) construction of models and parametric statistical methods, (3) credibility, and (4) simulation. A more detailed description of actuarial models (1) includes generating functions, classifying distributions, methods of creating new distributions, aggregate loss models, frequency and severity models. In constructing models (2), students fit distributions to data, learn to use modified data (such as censored, truncated, or grouped data), and select and parameterize models. The course compares and contrasts Classical, Bayesian, and Bühlmann credibility models. Using simulation (4), students simulate results from distributions and aggregate results.

There will be an emphasis on applications and computer-based implementation and assignments.

Note: Required text is either of the two textbooks listed below.

Text: Loss Models: From Data to Decisions, 3rd or 4th edition, Klugman, Panjer, and Willmot
ISBN: 978-1-118-31532-3 / 978-0-470-18781 Required

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525 Probability Theory — Yu — TTh 10 - 11:30AM

Prerequisites: *MATH 451 (strongly recommended). MATH 425/STATS 425.*

Course Description: The aim of this course is to provide mathematically rigorous basics of probability and some useful results on the subject. Some applications will be given through examples and exercises. There is substantial overlap with Math 425 (Intro. to Probability), but here more sophisticated mathematical tools are used and there is greater emphasis on proofs of major results. Math 451 is preferable to Math 450 as preparation, but either is acceptable. This course is a core course for the Applied and Interdisciplinary Mathematics (AIM) graduate program.

Text: Probability and Random Processes Third edition, Oxford, 2001, G. Grimmett and D. Stirzaker Required
Introduction to Probability Models Tenth Edition S.M. Ross ISBN:9780123756862 Optional

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526 Discrete stochastic processes — Nadtochiy TTH 10:00am-11:30am

Prerequisites: *Probability Theory 525 or equivalent courses* — Fahim TTh 11:30am-1pm

Course Description: The theory of stochastic processes is concerned with systems which change in accordance with probability laws. It can be regarded as the "dynamic" part of statistic theory. Many applications occur in physics, engineering, computer sciences, economics, financial mathematics and biological sciences, as well as in other branches of mathematical analysis such as partial differential equations. The purpose of this course is to provide an introduction to the many specialized treatise on stochastic processes. Most of this course is on discrete state spaces. It is a second course in probability which should be of interest to students of mathematics and statistics as well as students from other disciplines in which stochastic processes have found significant applications. Special efforts will be made to attract and interest students in the rich diversity of applications of stochastic processes and to make them aware of the relevance and importance of the mathematical subtleties underlying stochastic processes.

Text: Essentials of Stochastic Processes 2nd, Richard Durrett, Required

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555 Introduction to Complex Variables Viswanath MW 8:30-10:00

Course Description: This course is an introduction to the theory of functions of one complex variable which is

suitable for advanced undergraduate students and beginning graduate students in engineering, physics, and applied mathematics. While applications to physical problems are of primary importance, the arguments are appropriately rigorous, and students will be expected to prove identities and propositions as well as perform calculations. Topics covered include the calculus of functions of one complex variable, integrals and series, residue theory and its application to the evaluation of real integrals, and (if time permits), conformal mapping with applications to fluid flow as well as applications in approximation theory.

Text: Applied Complex Variables , John W. Dettman 978-0486646701 Optional

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557 Methods of Applied Mathematics II: Asymptotic Analysis Miller TTh 10:00am - 11:30am

Prerequisites: *Strong background in differential equations (e.g. Math 404), linear algebra (e.g. Math 419), and advanced calculus or real analysis (e.g. Math 451). Even more important is technical skill in complex variables and analysis at the level of Math 555 or Math 596.*

Course Description: Asymptotic analysis is the quantitative study of approximations. The fundamental idea is that one tries to solve a problem in applied mathematics (say, a boundary-value problem for a partial or ordinary differential equation) by embedding it into a family of problems with a parameter. Frequently the parameter has obvious physical meaning, e.g. fluid viscosity or aspect ratio. If the problem can be solved exactly for one special value of the parameter, then asymptotic analysis can be used to analyze how the solution changes as the parameter is tuned from the special value to a more physically reasonable one. This idea turns out to be meaningful even in many important cases where the problem has no solution at all for the special parameter value, leading to so-called singular perturbation theory.

The course will develop the general theory of asymptotic expansions, which are a kind of series in the perturbation parameter that are extremely useful in practice, in a way that is mathematically completely rigorous, despite the strange fact that they frequently fail to converge at all! We will then study how to use asymptotic expansions to evaluate integrals that cannot be computed in closed form and that are also difficult to approximate numerically. Next, we will turn to differential equations and use asymptotic expansions to evaluate solutions near certain singular points and also to study the way that solutions depend on parameters. At the end of the course we will study how the differential equations of diverse physical phenomena can be reduced, with the help of asymptotic expansions, to certain universal model equations that show up again and again in applied mathematics.

Specific applications covered: the small-viscosity theory of shock waves, the theory of quantum mechanics in the semiclassical limit, aspects of the theory of special functions, vibrations in nonlinear lattices, and surface water waves. Other applications as time permits.

Mathematical topics covered: Landau order notation, asymptotic expansions and asymptotic series, techniques for assigning finite sums to divergent asymptotic series, asymptotic expansions of integrals including the methods of Laplace, steepest descent, and stationary phase. Airy's equation and Airy functions. Linear differential equations in the complex domain and asymptotic behavior of solutions near singularities, including regular and irregular singular points, and the Stokes phenomenon. Regular perturbation theory for differential equations. Singular perturbation theory, including the WKB method and the methods of Liouville-Green and Langer. Singularly perturbed boundary-value problems, boundary and internal layer theory and matched asymptotic expansions. Perturbation theory of simple and degenerate eigenvalue problems. Parametric resonance and Mathieu's equation. The method of multiple scales. Weakly nonlinear oscillations. Canonical models of nonlinear waves.

Evaluation: Students will be evaluated on the basis of several homework sets, a research project culminating in an in-class presentation, and class participation and involvement. There are, however, no exams in Math 557.

Text: Applied Asymptotic Analysis , Miller, Peter 0-8218-4078-9 Required

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563 Advanced Mathematical Methods for the Biological Sciences — Forger — TTH 10-11:30am

Prerequsities: Familiarity with linear algebra (e.g. as in Math 217, 417, or 419), differential equations (e.g. as in Math 216, 286, or 316) and to have a basic familiarity with partial differential equations as would be gained by the completion of Math 450 or 454 or to have permission of the instructor.

Many processes within the body are complex. Mathematical models can help understand these processes by piecing together diverse data, determining underlying principles and predicting future behavior. The goal of this course will be to teach students how to take real biological data, convert it to a system of equations and simulate and/or analyze these equations. All theory will be presented in the context of trying to understand specific processes within the human body. Models and analysis will be presented in three main topics: Computational Neuroscience, Cell Biology and Fluid Flow in the Human Body. Subtopics include: computation within a single neuron, visual signal processing in the retina and brain, reaction-diffusion and stochasticity in genetic networks, propagation of pulses in arteries, “heart attacks” and the spread of urinary tract infections. Models will typically use partial differential equations. Consideration will be given in the problem sets and course project to interdisciplinary student backgrounds.

Text: Mathematical Physiology, Keener and Sneyd, Required

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566 Introduction to Algebraic Combinatorics — Stembridge — MWF 11am-12:00pm

Prerequisites: *Basic group theory and abstract linear algebra.*

Course Description: This course will be an introduction to algebraic combinatorics. Previous exposure to combinatorics will not be necessary, but experience with proof-oriented mathematics at the introductory graduate or advanced undergraduate level will be needed.

Most of the topics we cover will be centered around enumeration and generating functions. But this is not to say that the course is only about enumeration--counting formulas are often manifestations of of deeper structure.

Highlights along the way should include sieve methods, combinatorial factorization, Lagrange inversion, the permanent-determinant method, tableaux and plane partitions, the transfer matrix method, and the categorical gestalt for exponential generating functions.

Text: Enumerative Combinatorics, Vol. I 2nd, Richard Stanley 1107602629 Required

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567 Introduction to Coding Theory — Linowitz — MWF 10-11:00am

This course is an introduction to coding theory, a rather young area of mathematics which traces its roots to a 1948 paper of Claude Shannon. The main idea of coding theory is as follows. Suppose that we wish to transmit a message and know that our message is likely to be altered during the process of transmission due to factors such as weak signals and noise contamination. The goal of coding theory is to ensure that our original message can be recovered by the intended recipient.

After reviewing topics from linear algebra, abstract algebra and the theory of finite fields, we will develop the mathematical background required to study error-correcting codes. Specific topics to be covered include: Shannon’s Noisy Channel Coding Theorem, basic examples of codes (e.g., Hamming codes, BCH codes, cyclic codes, Reed-Muller codes and Reed-Solomon codes), weight enumerator polynomials and the Mac-Williams Sloane identity. If time permits we will cover asymptotic parameters, bounds and codes arising from algebraic geometry.

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572 Numerical Methods for Scientific Computing II Viswanath TTH 11:30-1:00

Prerequisites: *Solid background in advanced calculus, linear algebra and working knowledge of a computing programming language (such as C, C++ or Fortran) or a computing environment (such as Matlab or Python).*

Course Description: This course is an introduction to numerical methods for boundary-value and initial-value problems. The course will cover numerical methods for ordinary differential equations and for elliptic, parabolic and hyperbolic partial differential equations.

The course will focus on the derivation of methods, on their accuracy, stability and convergence properties with brief comments on practical aspects of efficient implementation of the methods. The course should be useful to students in mathematics, physics and engineering.

Contents:
Preliminaries (Eigenvalues, polynomial interpolation, quadrature). Runge-Kutta methods.

Multistep methods. Stiff problems and BDF methods. Accuracy, stability, convergence. The leapfrog method in molecular dynamics. Heat equation, finite differences, stability, accuracy. The Lax equivalence theorem. Advection equation, Lax-Wendroff, Lax-Friedrichs, the CFL stability condition. The transport equation. Numerical boundaries for the wave equation. Conservation laws and shocks. Finite volume methods. Introduction to finite element methods. Introduction to Discontinuous Galerkin methods (if time permits).

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582 Introduction to Set Theory Blass MWF 12 – 1:00pm

Prerequisites: *The official prerequisite, "Math 412 or 451 or equivalent experience with abstract mathematics," means that students should be comfortable with writing mathematical proofs.*

Course Description: This is an introductory course in axiomatic set theory.

Topics include:

The intuitive concept of set; paradoxes.
Type theory and the cumulative hierarchy of sets.
The Zermelo-Fraenkel axioms for set theory and basic set-theoretic constructions.
Set-theoretic representation of the fundamental concepts of mathematics (e.g., function, number) and proofs of basic properties of these concepts (e.g., mathematical

induction).
Infinite cardinal and ordinal numbers and their arithmetic.
The axiom of choice and equivalent axioms (e.g., Zorn's Lemma).

Additional topics may be discussed if time permits.

Text: Elements of Set Theory either edition is OK, Herbert B. Enderton Required

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590 An Introduction to Topology — Scott — MWF 12:00 - 1:00pm

Prerequisites: *Math 451 or an equivalent real analysis course.*

Course Description: This is an introduction to topology with the emphasis on the set-theoretic aspects of the subject.

Basic topics to be covered include metric spaces, topological spaces, continuous functions, homeomorphisms, compactness and connectedness. Many examples will be discussed including surfaces and manifolds. If time permits, some more advanced topics will also be covered. These could include quotient spaces, group actions, and a topological proof of the fundamental theorem of algebra.

Text: Topology, Second edition, James R. Munkres 0-13-181629-2 Required

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592 Introductory Algebraic Topology — Kriz — MWF 10-11:00am

Prerequisites: *The student should know the basic concepts of point set topology, i.e. topological spaces, continuous maps, homeomorphism, subspaces, the quotient topology, and compactness.*

Course Description: This is the course in which one learns how to prove rigorously that the sphere is not homeomorphic to the surface of a donut. The relevant tools are called topological invariants. We treat two main topological invariants, the fundamental group and singular homology, and related topics. Related topics include covering spaces, free groups and the Seifert-Van Kampen theorem and the homology of simplicial complexes and CW complexes, and applications to geometric questions such as the one mentioned above. Some necessary language of category theory will also be discussed. Time permitting, the Euler characteristic and Lefschetz trace formula (topological case) may also be covered.

Text: A Concise Course in Algebraic Topology , J.P. May 978-0226511832 Optional
Elements of Algebraic Topology J.R. Munkres 978-0201627282 Optional

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594 Algebra II — Speyer — MWF 2:00pm - 3:00pm

Prerequisites: *Prior exposure to the definitions of groups, rings, modules and fields. Abstract linear algebra over arbitrary fields. PID's and unique factorization in Z and k[x]. 513 and 593 are certainly enough; please talk to me if you have questions about your background.*

Course Description: Modern abstract algebra began with Galois's study of the roots of polynomial equations, and their groups of symmetries. This course will study finite groups and their representations, and will then focus on the beautiful subject of Galois theory -- connecting group computations with the behavior of roots of equations. Ideally, there will be a bit of number theory at the end of the course.

Text: Abstract Algebra 3, Dummit and Foote 978-0471433347 Required

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597 Analysis II (Real Analysis) — Barrett — MWF 12:00pm - 1:00pm

Prerequisites: *Math 451; Math 490 or 590 strongly recommended*

Course Description: This is one of the core courses for the mathematics doctoral program. Topics will include: Lebesgue measure on the real line and in R^n; general measures; measurable functions; integration; monotone convergence theorem; Fatou's lemma; dominated convergence theorem; product measures; Fubini's theorem; function spaces; Holder and Minkowski inequalities; functions of bounded variation; differentiation theory; Fourier analysis. Additional topics such as Hausdorff dimension and Sobolev spaces to be covered as time permits.

Text: An Introduction to Measure Theory , Terence Tao 978-0821869192 Required
An Epsilon of Room, 1: Real Analysis Terence Tao 978-0821852781 Required

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604 Complex Analysis II: Complex Dynamics — Jonsson — MWF 1:00pm-2:00pm

Prerequisites: *The course is suitable for graduate and advanced undergraduate students. The prerequisite is a course in complex analysis at the level of Math 596. Please note that this course is repeatable (once) for credit: you may sign up even if you have taken Math 604 once before.*

Course Description: This is a course on complex dynamics in one dimension, a subject often included in the somewhat imprecise notion of "Chaos Theory". We will study iterations of rational functions on the Riemann sphere. In particular we will study the partition of the

Riemann sphere into the Fatou and Julia sets, where the dynamics are "tame" and "chaotic", respectively. We will also study periodic points and equidistribution problems.

Depending on time and the interest of students, we may then study topics such as the Mandelbrot set or ergodic properties of rational maps. Alternatively, we may leave the complex world and instead focus on p-adic dynamics.

Text: Dynamics in One Complex Variable 3, John Milnor 978-0691124889 Required
Complex Dynamics 1 L. Carleson and T. Gamelin 978-0387979427 Optional

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612 Lie Algebras and Their Representations — Fulton — TT 11:30 - 1:00

Prerequisites: *Math 593, 594*

Course Description: This course studies finite-dimensional representations of complex semisimple Lie algebras, emphasizing the classical examples (the special linear, orthogonal, and symplectic Lie algebras) and relations with compact Lie groups and linear algebraic groups. We will discuss root systems and the classification of semisimple Lie algebras. We will investigate explicit descriptions of and formulas for these representations, their dimensions, and multiplicities.

Text: Introduction to Lie Algebras and Representation Theory Springer, Graduate Texts, James E. Humphreys 0-540-90052-7 Required
Complex Semisimple Lie Algebras Springer Monographs in Mathematics J.-P. Serre 3-540-67827-1 Optional

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615 Commutative Algebra II: Homological and Computational Commutative Algebra — Erman — MWF 2:00pm-3:00pm

Prerequisites: *Math 614, or something similar.*

Course Description: This course will cover topics in homological and computational commutative algebra. The essential homological topics include: the Koszul complex, Cohen--Macaulayness, and the Auslander--Buchsbaum formula. The essential computational topics include: Grobner bases, free resolutions, and Castelnuovo--Mumford regularity. Further topics will depend on the interests of the class, and may include: syzygies in algebraic geometry, spectral sequences, or derived categories.

There will be a heavy emphasis on computing examples in Macaulay2.

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There is no required textbook. Eisenbud's "Commutative Algebra with a view towards Algebraic Geometry" is the best overall reference.

Text: Commutative Algebra: with a View Toward Algebraic Geometry , David Eisenbud 978-0387942698 Optional

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626 Stochastic Analysis and Control — Bayraktar — TTh 10-11:30

Prerequisites: *Math 625 or instructor's approval.*

Course Description: This is an advanced graduate course which will cover selected topic in stochastic analysis, stochastic control and mathematical finance.

Text: Optimal Stochastic Control, Stochastic Target Problems, and Backward SDE , Nizar Touzi 1461442850, Required
Brownian Motion and Stochastic Calculus 2nd edition, Ioannis Karatzas and Steve Shreve 387976558, Required

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632 Algebraic Geometry 2 — Smith — TTh 10-11:30am

Prerequisites: *Math 614, Math 631, and Math 592.*

Course Description: This course is a continuation of Math 631. We will continue discussing classical algebraic geometry, working in more and more formal scheme theory as naturally arises in the study of algebraic geometry of complex projective varieties. We will continue discussing invertible sheaves, divisors, and maps to projective spaces, and some intersection theory in the classical case. Other topics include abstract schemes and cohomology of quasi-coherent sheaves. The texts is Shafarevich I, II and Hartshorne.

Text: Basic Algebraic Geometry Vol I and II 2, Shafarevich 3540575545, Required
Algebraic Geometry, Hartshorne 387902449, Required

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636 Topics in DIfferential Geometry: Quantum field theory and renormalizations — Ruan — TTh 11:30am-1:00pm

Prerequisites: *Some basic differential geometry. If you don't have it, this is fine too. We can also cover it in the class.*

Course Description: One of the most mysterious part of quantum field theory is renormalization. One needs it to make sense of path integral. On the other hand, it seems tobe so arbitrary to mathematician. Recently, Kevin Costello has put a large part of it in the formal setting. His key idea is to interpret path integral as certain cohomology class. Then, a different choice of renormalization scheme becomes a homotopy. Right now, this new formulism has already yielded many applications. In this course, we will not assume any familiarity to quantum field theory. The first half of the course will be a gentle introduction to quantum field theory. In the second half, we will try to cover part of this book "Renormalization and effective field theory".

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651 Waves and Imaging in Random Media Schotland MW 4-5:30pm

Prerequisites: *Basic partial differential equations; some knowledge of probability theory would be useful, but not essential.*

Course Description: This is a special topics course. The focus is on the theory of wave propagation in inhomogeneous media in various asymptotic regimes including: (i) geometrical optics of high-frequency waves (ii) homogenization of low-frequency waves in periodic and random media (iii) radiative transport and diffusion theory for high-frequency waves in low-frequency random media. Applications to inverse problems in imaging will be considered. The necessary tools from asymptotic analysis, scattering theory and probability will be developed as needed. The course is meant to be accessible to graduate students in mathematics, physics and engineering.

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654 Introduction to Mathematical Fluid Dynamics Doering MW 8:30 - 10:00am

Prerequisites: *Elementary physics, Newtonian mechanics, elementary dynamical systems theory, differential equations, complex analysis, and computer literacy as expected of mathematically minded science and engineering graduate students.*

Course Description: Course content: Derivation of the Euler and Navier-Stokes equations; Conservation laws for mass, momentum, and energy; stream functions, vorticity, point vortices; potential flow past a bluff body; viscous flow; introduction to hydrodynamic instabilities; introduction to turbulence; existence and regularity of solutions to the Navier-Stokes equations and the Clay Prize problem.

Audience: Graduate students from mathematics, the physical sciences and engineering looking for an introduction to mathematical fluid dynamics.

Work expected: Attend lectures and complete several problem sets that will be assigned, collected & marked during the term. Grades based on homework and class participation.

Subsequent Courses: Math 655 Topics in Fluid Dynamics

Text: Applied Analysis of the Navier-Stokes Equations latest, Doering & Gibbon 052144568X Required
A Mathematical Introduction to Fluid Mechanics latest, Chorin & Marsden, 387979182, Optional

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657 Nonlinear Partial Differential Equations Bieri TTh 2:30-4:00pm
The Mathematics of General Relativity Theory

Prerequisites: *Some knowledge in partial differential equations and differential geometry.*

Course Description: The theory of general relativity (GR) unifies space, time and gravitation. The fundamental objects of study in GR are the spacetimes, which may have a rich geometric structure. A spacetime in general relativity theory is a Lorentzian manifold where the metric solves the Einstein equations.

These equations can be thought of as the laws of GR. They can be written as a system of nonlinear, second-order, hyperbolic pde. The unknown is the metric. Typical physical questions are formulated as initial value problems for the Einstein equations under specific conditions. The solution will lay open the geometry of the resulting spacetime. GR is a rich interplay between geometry, analysis and physics. Here, mathematical results have direct implications in physics. Today, the methods of geometric analysis have proven to be most effective to investigate these structures. Phenomena like the stability of galaxies, the formation of black holes or even the fate of our Universe are described by solutions of the Einstein equations. Gravitational waves are investigated by means of Lorentzian geometry.

This course shall focus on Lorentzian geometry and the Einstein field equations within general relativity. After brief reviews of special relativity and some fundamental facts from differential geometry (Lorentzian/Riemannian manifolds, curvature, Bianchi identity, etc.), we will introduce the spacetime in GR. Null hypersurfaces in Lorentzian manifolds will be treated. We shall then discuss the Schwarzschild solution and black holes. In view of the latter, we shall prove Penrose's incompleteness theorem and study the extensions of this result by Hawking and Penrose. Those results are better known as the `singularity theorems'. The most recent breakthrough (2008) along this way is certainly Christodoulou's result on the formation of black holes, showing that a closed trapped surface will form through the focusing of gravitational waves. Finally, as time allows, we will investigate gravitational waves. In particular, we will discuss nonlinear effects of these waves. In order to establish these results a combination of geometric and analytic methods will be introduced.

Text: General Relativity, Univ.Chicago Press, 1984, Robert Wald Optional

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669 Topics in Convexity Barvinok TTh 1:00 - 2:30 pm

Prerequisites: *Good knowledge of linear algebra*

Course Description: Convexity is very easy to define formally and to understand intuitively. It plays an important role in number theory, algebraic geometry, analysis, optimization, control theory and mathematical economics. The goal of this course is to demonstrate the unifying power of convexity on a variety of applications, chosen from the following topics.

1. Classical structural results: Theorems of Caratheodory and Helly, their generalizations, separation theorems and the Krein-Milman Theorem, their applications in analysis, combinatorics and optimization. Theorems of Birkhoff - von Neumann and Schur - Horn.

2. Positive semidefinite convexity. The structure of the cone of positive semidefinite matrices, applications to metric embedding problems and to quadratic convexity theorems of Brickman and Toeplitz-Hausdorff. Applications of semidefinite programming to codes, sphere packing, graph coloring, connections to Lovasz' theta-function and Grothendieck's inequality.

3. Hyperbolic polynomials and associated convex cones. The recent proof by Gurvits of a far-reaching generalization of the van der Waerden conjecture for permanents.

Some of the more classical material can be found in "A Course in Convexity" by A. Barvinok. For newer developments, lecture notes will be posted.

Grading: we will have a number of homework problem sets.

Text: A Course in Convexity 2002, Alexander Barvinok 0-8218-2968-8 Optional

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671 Numerical Methods I: Fast algorithms Veerapaneni TTh 10:00-11:30am

Prerequisites: *Basic theory of ODEs/PDEs, linear algebra and programming experience. However, we will provide a short review on the necessary background.*

Description: This course will survey modern fast numerical algorithms with emphasis on solvers for partial differential equations. Despite the rapid advances in computing capabilities, numerous computational problems that arise in sciences and engineering are intractable without optimal numerical solvers. In this course, we will learn how to identify computational bottlenecks and apply suitable techniques for resolving them. Hands-on examples and case studies from various disciplines will be part of the course.

Grading: The grade will be entirely based on 3 or 4 projects and class participation. For the most part, students can chose projects from their own research areas as well as suggest topics for group projects.

Topics :

- Complexity analysis in scientific computing
  Review of linear & nonlinear solvers
  Krylov subspace methods
- Cartesian grid methods
  Multigrid and multiresolution methods
- Fourier spectral methods
  Nonuniform FFTs
  Butterfly algorithms
  Ewald summation
- Adaptive algorithms
  Tree codes
  Fast multipole methods
  Fast Gauss transform

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684 Recursion Theory — Blass — TTh 10 - 11:30am

Prerequisites: It is useful to have had a first course in mathematical logic, but it is not strictly necessary if you are willing to take some basic facts on faith.

Course Description: This course is about two fundamental concepts of mathematics: computability and provability. The first topic to be covered is a precise definition of computability in principle, where "in principle" means that a computation must give a result after finitely many steps but we place no specific bounds on the number of steps or the amount of memory used. In fact, several precise definitions of computability have been proposed, but they all turn out to be equivalent. We shall discuss a few of them and develop the tools needed to prove their equivalence. We shall also discuss the connection between computability and definability, giving specific examples of definable but uncomputable functions.

The second part of the course, building on the concepts and techniques of the first part, explores the limits of provability in axiomatic systems. It includes the proof of Goedel's incompleteness theorems --- that reasonable axiomatic systems cannot exactly capture truth, even in the limited domain of the arithmetic of natural numbers, and that they cannot prove their own consistency.

The final part of the course is a deeper study of computability, and in particular a classification of uncomputable functions according to a criterion of the form "if we had access to values of f then we could compute values of g." The analysis of this relation has led to some deep andelegant arguments, the simplest of which will be covered in this course.

Text: Fundamentals of Mathematical Logic , Peter G. Hinman Optional

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697 Topics in Topology: Teichmuller theory and its generalizations — Canary — MWF 2:00pm - 3:00 pm

Course Description: If S is a closed orientable surface of genus at least 2, the Teichmuller T(S) of S is the space of isometry classes of marked hyperbolic surfaces homeomorphic to S. Equivalently, we may think of T(S) as the space of conformal classes of Riemann surfaces which are homeomorphic to S. The study of Teichmuller space arises naturally in many fields. For example, one may view Teichmuller space as the ``universal cover'' of Moduli space, since T(S) is homeomorphic to an open ball and the mapping class group acts properly discontinuously on T(S) so that we may identify its quotient with Moduli space.

We will assume only material from the alpha courses, so we will begin with a basic introduction to hyperbolic geometry. We will produce parameterizations of Teichmuller space from both geometric and complex analytic viewpoints. We will also discuss dynamics on hyperbolic surfaces, e.g. the ergodicity of the geodesic flow.

We will also, as time permits, study two natural generalizations of Teichmuller space. First, we will study spaces of marked hyperbolic 3-manifolds homotopy equivalent to S, e.g. quasifuchsian space. Second, we will study the Hitchin component of the space of representations of the fundamental group of S into PSL(n,R). The exact topics covered will depend on the interests of the students. Moreover, we will give a topological viewpoint on the Deligne-Mumford compactification of Moduli space

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710 Modern Analysis: Introduction to Harmonic Analysis — Wu — TTh 1-2:30pm

Prerequisites: *Math 597, Math 596*

Course Description: Math 710 Introduction to Harmonic Analysis Harmonic analysis is a beautiful and core subject in mathematics, and an important tool in the modern study of partial differential equations. The goal of this course is to introduce students to some basic methods and tools in treating nonlinear operators. Topics covered include the Fourier transform, the Calderon-Zygmund operators, A_p weights, hardy spaces, BMO, Carleson measure, para-products, the T1Theorem and the Cauchy integral of Calderon. If time permits, we will discuss applications in Partial Differential Equations.

Course materials will be taken from the following references and journal articles.
References:
Jean-Lin Journe: Caldeon-Zygmund operators, pseudo-differential operators, and the Cauchy integral of Calderon.
Coifman & Meyer: Wavelets: Calderon-Zygmund and multilinear operators.
Elias M. Stein: Harmonic analysis: real-variable methods, orthogonality, and oscillatory integrals.

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732 Introduction to birational geometry — Mustata — MWF 12:00-1:00PM

Prerequisites: *Familiarity with algebraic varieties, as covered in Math 631, as well as a working knowledge of sheaves and cohomology, at the level of Serre's FAC.*

Course Description: A recent breakthrough in the study of higher-dimensional algebraic varieties has been the proof of the finite generation of the canonical ring by Birkar, Cascini, Hacon, and McKernan. A little later, Cascini and Lazic gave a different proof of the same result that is more elementary, in the sense that it does not use the machinery of the Minimal Model Program or fancy extension theorems. On the other hand, once finite generation is settled, the known results from Mori theory (including the recent ones proved by BCHM) follow. The goal of the course is to give an introduction to the basic techniques from birational geometry, cover the Cascini-Lazic proof of the finite generation of the canonical ring, and explain how this implies the main results in the Minimal Model program.

The following is a rough outline of the course:

1. Intersection numbers of divisors on complete varieties. The theorems of Nakai-Moishezon and Kleiman.
2. Basic properties of nef and big divisors.
3. The Kodaira and Kawamata-Viehweg vanishing theorems.
4. Singularities of pairs (log terminal and log canonical singularities, centers of log canonical singularities, the log canonical threshold).
5. Asymptotic base loci of line bundles and asymptotic orders of vanishing.
6. The finite generation of the canonical ring, after Cascini-Lazic.
7. The Minimal Model program (after Keel-Hu and Corti-Lazic).

Text: Birational geometry of algebraic varieties , Janos Kollar, Shigefumi Mori, Optional
Positivity in algebraic geometry, Robert Lazarsfeld, Optional

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776 Class Field Theory Lagarias MWF 1:00pm-2:00pm

Prerequisites: *Some familiarity with algebraic number fields, and with p-adic fields.*

Course Description: Class Field Theory concerns the description of properties of abelian Galois extensions of a field k in terms of invariants of k. Global class field theory concerns k a number field or function field over finite field, and the invariants of k are (ray) class groups and units. Local class field theory concerns k a one-dimensional local field, such as a p-adic field, and invariants are field elements and norm groups. These two theories embody the GL(1) part of the (local and global) Langlands program. The course aims to cover both the ideal theory version and idelic version, and at end their relation to Langlands program. At present I plan to follow Milne's book (notes) and start with local class field theory (but this could change).

Text: Class Field Theory , J. Milne, Optional