1. Let $f : X \to Y$ be a continuous map of metric spaces. The limit set $L(f)$ of $f$ is the set of $y$ in $Y$ such that $y = \lim f(x_n)$ for some sequence $(x_1, x_2, \ldots)$ in $X$ with no convergent subsequence.

a) Show that $f(X)$ is closed in $Y$ if and only if $L(f) \subset f(X)$.

b) Give an example of a continuous $f : \mathbb{R} \to \mathbb{R}^2$ with $f(\mathbb{R})$ closed and $L(f) \neq \emptyset$.

2. Let $X$ be the complement of two circles $\{x^2 + y^2 = 1, z = 1\}$ and $\{x^2 + y^2 = 1, z = -1\}$ in $\mathbb{R}^3$. Show that $X$ is path connected and determine $\pi_1(X)$ and $H_*(X)$.

3. Let $f : S^2 \to \mathbb{R}$ be given by $f(x, y, z) = xy + z^2$. Find the critical points of $f$.

4. Let $K$ be the simplicial complex consisting of the 3-simplices $(v_1, v_2, v_3, v_4)$, $(v_3, v_4, v_5, v_6)$, $(v_1, v_2, v_5, v_6)$ and their faces. (All the $v_k$ are distinct.) Compute $H_*(K)$.

5. Prove or disprove:

a) The space $X$ has the discrete topology if and only if for each $x$ in $X$, the connected component of $x = \{x\}$.

b) $\prod_{n=1}^{\infty} X_n$ has the discrete topology if and only if $X_n$ has the discrete topology for each $n$. 
1. a) Let $\Delta \subset S^n \times S^n$ be the diagonal subspace; $\Delta = \{(x, x) | x \in S^n\}$. Show that the projection $p : S^n \times S^n - \Delta \rightarrow S^n$ given by $p(x, y) \rightarrow x$ is a homotopy equivalence.

b) Let $T^3 = S^1 \times S^1 \times S^1$ and let $\Delta \subset T^3 \times T^3$ be the diagonal subspace.

Compute $\pi_1(T^3 \times T^3 - \Delta)$.

c) Show that the projection $p : T^3 \times T^3 - \Delta \rightarrow T^3$ given by $p(x, y) \rightarrow x$ is not a homotopy equivalence.

2. a) Let $A$ be a bounded subset of $\mathbb{R}^n$ and $f : A \rightarrow \mathbb{R}$ a uniformly continuous map. Must $f$ be bounded? Give a counterexample or a careful proof.

b) Let $A$ be a bounded subset of $l_2$ and $f : A \rightarrow \mathbb{R}$ a uniformly continuous map. Must $f$ be bounded? Give a counterexample or a careful proof.

(Recall that $l_2 = \{(x_1, x_2, \ldots) | \sum_{n=1}^{\infty} x_n^2 < \infty\}$; $l_2$ is endowed with the metric $d((x_1, x_2, \ldots), (y_1, y_2, \ldots)) = \sqrt{\sum_{n=1}^{\infty} (x_n - y_n)^2}$.)

3. Describe the set of homotopy classes of maps from $(S^1 \times S^2, p)$ to $(S^1, 1)$ and prove your answer. Recall that $S^1 = \{z \in \mathbb{C} | z \bar{z} = 1\}$. Here $p = (1, *)$ for some point $*$ in $S^2$.

4. Let $X_q = \{(x, y, z, w) \in \mathbb{R}^4 | x^2 - y^2 + w = q, \ z^2 + w^2 = 1 \}$. For what values of $q$ is $X_q$ a manifold?

5. Let $X = (S^1 \times S^1)/\sim$ where $(x, y) \sim (y, x)$. Compute $H_\ast(X)$. 