1. Use the method of residues to calculate the integral
\[ \int_0^\infty \frac{\sin x}{x} \, dx \]
Show all estimates.

2. Assume that \( f \in L^\infty(E) \), where \( m(E) < \infty \). Prove that
\[ \lim_{p \to \infty} \| f \|_{L^p(E)} = \| f \|_{L^\infty(E)}. \]

3. What is the value of
\[ \int_c \frac{32z^7 + 3z^2}{4z^8 + \sqrt{z^4 + 2}} \, dz, \]
where \( c \) is the positively oriented unit circle.

4. Assume that \( f, f_k \in L^2(R^n) \), for \( k = 1, 2, \ldots \) and
\[ f_k \to f \quad \text{a.e. in } R^n, \quad \| f_k \|_{L^2(R^n)} \to \| f \|_{L^2(R^n)}, \quad \text{as } k \to \infty. \]
Show that \( f_k \) converge to \( f \) in \( L^2(R^n) \).

5. True or false. Give a short explanation.
   a. There exist a function \( f \) analytic in the unit disc \( D \) such that
   \[ f(0) = 0, \quad f\left(\frac{1}{2}\right) = \frac{1}{10}, \quad \text{and } \text{Re}\{ f(z) \} \geq 0, \text{for all } z \in D. \]
   b. One can find a function \( g \) that is analytic in \( \{ z \in C : 0 < |z| < \epsilon \} \), \( g \) has an essential singularity at 0 and \( \frac{1}{g(z) - 1} \) is bounded.
   c. If \( h \) is analytic in \( \Omega \) and \( \gamma \) is a simple closed curve in \( \Omega \), then \( \int_\gamma h(z) \, dz = 0. \)
   d. Suppose \( f \) is entire and \( |f(z)| \leq 1 + |z|^{1/2} \) for all \( z \), then \( f \) is a constant.
1. Assume that $f \in C^1[0, \pi]$, and \( \int_0^\pi f(x) \, dx = 0 \). Show that there exists a constant $C$, independent of $f$, such that
\[
\int_0^\pi |f(x)|^2 \, dx \leq C \int_0^\pi |f'(x)|^2 \, dx.
\]

2. Find a conformal map from \( D = \{ z \in \mathbb{C} : |z| < 1 \} \setminus \{ iy : \frac{1}{3} \leq y \leq 1 \} \) to \( \Omega = \{ z : 1 < \Re z < 2 \} \).

3. Let \( (X, \mathcal{M}, \mu) \) and \( (Y, \mathcal{N}, \nu) \) be \( \sigma \)-finite measure spaces and let \( K(x, y) \) be measurable with respect to the product \( \sigma \)-algebra \( \mathcal{M} \times \mathcal{N} \). Assume that there is a constant $A$, such that for all \( x \in X \),
\[
\int_Y |K(x, y)| \, d\nu(y) \leq A,
\]
and for all \( y \in Y \),
\[
\int_X |K(x, y)| \, d\mu(x) \leq A.
\]
Let \( 1 \leq p \leq \infty \), and for \( f \in L^p(X, \mathcal{M}, \mu) \), define
\[
T(f)(y) = \int_X K(x, y) f(x) \, d\mu(x).
\]
Show that
\[
\|T(f)\|_{L^p(\nu)} \leq A \|f\|_{L^p(\mu)}.
\]

4. a. Suppose \( \Omega \) is a domain, \( D \) is a disc such that \( \overline{D} \subset \Omega \), \( f \) is analytic in \( \Omega \), \( f \) is not constant and \( |f| \) is constant on the boundary of \( D \). Prove that \( f \) has at least one zero in \( D \).

b. Find all entire functions \( f \) such that \( |f(z)| = 1 \) when \( |z| = 1 \).
5. Let $f_n : R \to [0, \infty)$ be nondecreasing for each natural number $n$. Assume that for all $x \in R$,

$$f(x) = \sum_{n=1}^{\infty} f_n(x) < \infty.$$ 

Show that

$$f'(x) = \sum_{n=1}^{\infty} f'_n(x) \quad a.e. \ x \in R.$$