1. Let $Z$ be a convex 10-gon in the plane with vertices $A_0, A_1, A_2, A_3, A_4, B_4, B_3, B_2, B_1, B_0$ appearing in this order on the boundary (oriented counter-clockwise). Let $X$ be the topological space obtained from $Z$ by gluing the line segments $A_0A_1$ with $B_2B_3$, $B_0B_1$ with $A_2A_3$, $A_1A_2$ with $B_1B_2$, $A_3A_4$ with $B_3B_4$, $A_0B_0$ with $B_4A_4$. All pairs of line segments are attached by linear maps with the vertices corresponding in the order listed (first to first, last to last).

(a) Calculate $\pi_1(X)$.

(b) Classify the surface $X$.

2. Prove that every CW-structure on $\mathbb{R}P^n$ has at least one cell in each dimension $0, 1, \ldots, n$.

3. Let $X$ be a graph with one vertex and two edges. Does there exist a connected covering $f : Y \to X$ which is regular and a connected covering $g : Z \to Y$ which is regular such that $fg : Z \to X$ is not a regular covering? Prove your answer.

4. Let $Z = (\mathbb{C} \setminus \{e^{2k\pi i/5} \mid k \in \mathbb{Z}\}) \times [0,1]$. Let a space $Y$ be obtained from $Z$ by identifying $(z, 0)$ with $(ze^{2\pi i/5}, 1)$ for every $z \in \mathbb{C} \setminus \{e^{2k\pi i/5} \mid k \in \mathbb{Z}\}$. Compute $\pi_1(Y)$.

5. Let

\[ S^2 = \{(x, y, z) \in \mathbb{R}^3 \mid x^2 + y^2 + z^2 = 1\}, \]

\[ D^3 = \{(x, y, z) \in \mathbb{R}^3 \mid x^2 + y^2 + z^2 \leq 1\}, \]

Let $f : S^2 \to S^2$ be a (continuous) map of degree $k$, and let $\pi : S^2 \to \mathbb{R}P^2$ be a covering. Let $X$ be the pushout of the diagram

\[ \xymatrix{ S^2 \ar[r]^{\pi \circ f} & \mathbb{R}P^2 \ar[d] \cr & D^3. } \]

Calculate the homology of $X$. 