Syllabus for Math 556 - Fall 1995

Text


1. Fourier Series (chap. 2)

Fourier series of a periodic function, Bessel's inequality

*PC*(a, b), *PS*(a, b), Dirichlet kernel, pointwise convergence

uniform convergence, Gibbs phenomenon

differentiation of Fourier series, relation between smoothness of f and decay of $c_n$

sine and cosine series

separation of variables

\[ u_t = ku_{xx}, \quad 0 \leq x \leq l, \quad u(x, 0) = f(x), \quad u(0, t) = u(l, t) = 0 \]

\[ u_t = ku_{xx}, \quad 0 \leq x \leq l, \quad u(x, 0) = f(x), \quad u_x(0, t) = u_x(l, t) = 0 \]

\[ u_t = ku_{xx}, \quad x > 0, \quad u(0, t) = f(t) \text{ (heat conduction below Earth's surface)} \]

\[ u_{tt} = c^2 u_{xx}, \quad 0 \leq x \leq l, \quad u(x, 0) = f(x), \quad u_t(x, 0) = g(x), \quad u(0, t) = u(l, t) = 0 \]

(derive d'Alembert's formula from Fourier series)

2. Orthogonal Functions (chap. 3)

inner product, Cauchy-Schwarz inequality, orthogonality

completeness, characterization of an orthonormal basis for $L^2(a, b)$

dominated convergence theorem (statement only)

best approximation in $L^2(a, b)$

self-adjoint differential operators, regular Sturm-Liouville problems

spectral theorem (statement only)

examples

\[ f'' + \lambda f = 0, \quad f(-\pi) = f(\pi), \quad f'(-\pi) = f'(\pi) \]

\[ f'' + \lambda f = 0, \quad f'(0) = \alpha f(0), \quad f'(l) = \beta f(l) \]

3. Boundary Value Problems (chap. 4)

1-d heat flow

\[ u_t = ku_{xx}, \quad u(x, 0) = f(x), \quad u_x(0, t) = \alpha u(0, t), \quad u_x(l, t) = -\alpha u(l, t) \]

(Newton's law of cooling)

\[ u_t = ku_{xx}, \quad u(x, 0) = f(x), \quad u(0, t) = 0, \quad u(l, t) = A \text{ (inhomogeneous b.c.)} \]

\[ u_t = ku_{xx} + R, \quad u(x, 0) = 0, \quad u(0, t) = u(l, t) = 0 \text{ (radiation)} \]

1-d wave equation

\[ u_{tt} = c^2 u_{xx} + F, \quad u(x, 0) = f(x), \quad u_t(x, 0) = g(x), \quad u(0, t) = u(l, t) = 0 \]

\[ u_{tt} = c^2 u_{xx}, \quad u(x, 0) = f(x), \quad u_t(x, 0) = g(x) \]

\[ u_x(0, t) = u_x(l, t) = 0 \text{ (pipe open at 2 ends)} \]

\[ u(0, t) = u_x(l, t) = 0 \text{ (pipe open at 1 end, note effect on frequencies)} \]

Dirichlet problem for Laplace equation

\[ \Delta u = 0 \text{ in } D, \quad u = f \text{ on } \partial D \text{ (rectangle, annulus, disk, Poisson kernel for unit disk)} \]

multivariable Fourier series

2-d, 3-d heat flow on rectangular domains

electrostatic potential due to charge in a rectangular volume
4. Fourier Transform (chap. 7)
convolution, approximate identity
Fourier transform on $L^1$, Riemann-Lebesgue lemma, inversion formula
Fourier transform on $L^2$, Plancherel theorem
applications
heat kernel
Dirichlet problem on half-space, Poisson kernel
signal analysis, band-limited functions, sampling theorem
Heisenberg inequality
Sturm-Liouville problems on an unbounded domain, sine and cosine transform
$u_t = k u_{xx}, \ 0 < x < \infty, \ u(x,0) = f(x), \ u_x(0,t) = 0$ (interpretation via images)
multivariable Fourier transform
discrete Fourier transform, FFT

5. Generalized Functions (chap. 9)
test functions, $L^1_{loc}$
distributions, delta function
weak convergence, approximation by convolution
weak solution of differential equations
$u' = 0, \ xu' = 0, \ x^2 u' = 0$
$u_t + cu_x = 0$

6. Green's Functions (chap. 10)
construction of $G(x,y)$ for $Lu = a_2(x)u'' + a_1(x)u' + a_0(x)u, \ B_0u = B_1u = 0$
examples
$Lu = u''', \ u(0) = u(1) = 0$
$Lu = u''', \ u'(0) = u'(1) = 0$ (counterexample, state Fredholm alternative)
$Lu = u'' + \alpha^2u, \ u(0) = u(1) = 0$
$Lu = u'' - \alpha^2u, \ u(\pm \infty) = 0$
eigenfunction expansions for $G(x,y), \ \delta(x)$
fundamental solution of Laplace equation in 2-d, 3-d

Note: there are many more examples contained in the exercises for these chapters.

References
Fourier Series and Integrals H. Dym & H.P. McKean, Academic Press, 1972
Green's Functions and Boundary Value Problems I. Stakgold, John Wiley, 1979

submitted by Robert Krasny, 12/19/95