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<tr>
<td>Monday, November 07, 2022</td>
<td>3:00pm-4:15pm</td>
<td>RTG Seminar on Number Theory -- Jialiang Zou (UM) Spherical varieties and L-functions</td>
<td>4088 East Hall</td>
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<td>4:00pm-5:00pm</td>
<td>Complex Analysis, Dynamics and Geometry -- Rachel Greenfeld (IAS) Aperiodicity of</td>
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<td>Integrable Systems and Random Matrix Theory -- Bjorn Berntson (KTH) Nonlocal nonlinear</td>
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<td>Schrödinger equations and Calogero-Moser systems -- ZOOM ID: 926 6491 9790 Virtual</td>
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<td>4:00pm-5:00pm</td>
<td>Student Combinatorics -- Amanda Shwartz (UM) Dimers and Tilings</td>
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<td>4:30pm-5:30pm</td>
<td>Group, Lie and Number Theory -- Yujie Xu (MIT) Hecke algebras for p-adic groups and explicit</td>
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<td>Local Langlands Correspondence for G_2</td>
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<td>Tuesday, November 08, 2022</td>
<td>3:00pm-4:00pm</td>
<td>Student Commutative Algebra -- Teresa Yu (UM) Eagon-Northcott Complexes and Applications</td>
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<td>4:00pm-5:00pm</td>
<td>Colloquium Series -- Ziquan Zhuang (Johns Hopkins University/IAS, Princeton) Canonical</td>
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<td>metrics on Fano varieties</td>
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<td>5:00pm-6:00pm</td>
<td>Student Analysis -- Ethan Zell (University of Michigan) Mean Field Equilibria</td>
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<td>Wednesday, November 09, 2022</td>
<td>2:30pm-4:00pm</td>
<td>Learning Seminar in Algebraic Combinatorics -- Nir Gadish (University of Michigan)</td>
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<td>Producing a Poisson cluster variety using dimer models</td>
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<td>MCAIM Colloquium -- Philipp Schoenhoefer (University of Michigan Chemical Engineering)</td>
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<td>Hard Particle Self-Assembly From the Perspective of Geometric Frustration</td>
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<td>Financial/Actuarial Mathematics -- Ibrahim Ekren (Florida State University) Monge-Kantorovich</td>
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<td>Duality, Informed Trading, and Risk Aversion</td>
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<td>Student Arithmetic -- Simran Khunger () Vinogradov Mean Value Theorem</td>
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<td>4:00pm-5:20pm</td>
<td>Algebraic Geometry -- Ziquan Zhuang (Johns Hopkins University) Stable degenerations of</td>
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<td>4:00pm-5:30pm</td>
<td>RTG Seminar on Geometry, Dynamics and Topology -- Stephen Cantrell (U Chicago) Orbital</td>
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<td>4:00pm-5:30pm</td>
<td>Logic -- Patrick Lutz (UCLA) The Solecki dichotomy and the Posner Robinson theorem</td>
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<td>Thursday, November 10, 2022</td>
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<td>Commutative Algebra -- Hannah Klawa (George Mason University) Global perinormality in a</td>
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<td>Topology -- Allen Yuan (Columbia University) A brief tour of higher algebra</td>
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<td>Differential Equations -- Zhongshan An (UMICH Mathematics) Static vacuum metrics with</td>
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<td>Arithmetic Geometry Learning -- Andy Jiang (UM) The descent method</td>
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<td>Student Dynamics/Geometry Topology -- Schinella D'Souza (University of Michigan) Thurston's</td>
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<td>topological characterization of rational functions</td>
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<td>9:00am-10:00am</td>
<td><strong>Variational Analysis and Optimization</strong> -- Marco A. López (University of Alicante, Spain) A new tour on the subdifferential of the supremum function</td>
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<td>3:00pm-4:00pm</td>
<td><strong>Applied Interdisciplinary Mathematics (AIM)</strong> -- Jennifer Franck (University of Wisconsin) Predictive Modeling of Oscillating Foil Wake Dynamics -- Zoom Virtual</td>
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<td>3:00pm-3:50pm</td>
<td><strong>Student Algebraic Geometry</strong> -- Andy Gordon (Michigan) Algebraic Geometric Codes and Curves with Many Points</td>
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<td>3:00pm-4:00pm</td>
<td><strong>Combinatorics</strong> -- Paul Federbush (University of Michigan) Some Positivities for i-Matchings on r-Regular Bipartite Graphs and Lattices</td>
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<td>4:00pm-5:30pm</td>
<td><strong>Preprint Algebraic Geometry</strong> -- Lena Ji Geometric representability of 1-cycles on rationally connected threefolds</td>
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<td>4:00pm-5:30pm</td>
<td><strong>Special Events</strong> -- Jinho Baik &amp; Asaf Cohen (UM) Faculty Spotlight on Probability</td>
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RTG Seminar on Number Theory  
Monday, November 07, 2022, 3:00pm-4:15pm  
4088 East Hall  
Jialiang Zou (UM)  

Spherical varieties and L-functions

In this talk, we study the unramified spectrum of a homogeneous spherical variety X. We will discuss Sakellaridis's work on computing the eigenfunctions on spherical varieties under the action of the spherical Hecke algebra, which generalise the classical Casselman Shalika type formula. We will also discuss a variant of this formula, which involves the dual group of the spherical variety X and certain quotient of L-functions. As an application, we will present the unramified Plancherel formula for X.

Complex Analysis, Dynamics and Geometry  
Monday, November 07, 2022, 4:00pm-5:00pm  
3096 East Hall  
Rachel Greenfeld (IAS)  

Aperiodicity of translational tilings

Translational tiling is a covering of a space using translated copies of some building blocks, called the "tiles", without any positive measure overlaps. What are the possible ways that a space can be tiled? The most well known conjecture in this area is the periodic tiling conjecture, which asserts that any tile of $\mathbb{R}^d$ (or $\mathbb{Z}^d$) admits a periodic tiling. In a joint work with Terence Tao, we construct a counterexample to this conjecture. In the talk, I will survey the study of the periodicity of tilings and discuss our recent progress.
Integrable Systems and Random Matrix Theory  
Monday, November 07, 2022, 4:00pm-5:00pm  
ZOOM ID: 926 6491 9790 Virtual  
Bjorn Berntson (KTH)  
Nonlocal nonlinear Schrödinger equations and Calogero-Moser systems

The integrability and applicability of the nonlinear Schrödinger (NLS) equation have inspired the development of various integrability-preserving NLS relative equations. I will discuss a new such equation that interpolates between the mixed Manakov system and a known nonlocal NLS equation containing a Hilbert transform. This new system admits families of exact analytic solutions parameterized by the solutions of hyperbolic and elliptic Calogero-Moser many-body systems. Special attention is given to the second family of solutions, which features the interaction of periodic waves against a dynamic background. This is joint work with Alexander Fagerlund (KTH Physics).

A recording of the talk can be found [here](https://youtu.be/6Aj59WGs6ek).

Student Combinatorics  
Monday, November 07, 2022, 4:00pm-5:00pm  
3866 East Hall  
Amanda Shwartz (UM)  
Dimers and Tilings

The dimer model studies the set of perfect matchings on a graph. It has many applications in probability, discrete geometry, and combinatorics. I'll give an introduction to the dimer model and discuss connections to tilings with convex polygons.

Group, Lie and Number Theory  
Monday, November 07, 2022, 4:30pm-5:30pm  
4088 East Hall  
Yujie Xu (MIT)  
Hecke algebras for p-adic groups and explicit Local Langlands Correspondence for G_2

I will talk about my recent joint work with A.-M. Aubert (with special contribution by C.-C. Tsai) on the explicit Local Langlands Correspondence for G_2. This uses our earlier results on Hecke algebras attached to Bernstein components of reductive p-adic groups, as well as an expected property on cuspidal support due to AMS, along with a list of characterizing properties. In particular, we obtain "mixed" L-packets containing F-singular supercuspidals and non-supercuspidals.
Determinantal ideals are an important class of ideals that appear throughout commutative algebra, algebraic geometry, and representation theory. Eagon-Northcott complexes provide a way to study certain kinds of determinantal ideals (those generated by maximal minors). After reviewing Koszul complexes, we will introduce Eagon-Northcott complexes and their basic properties. We will see how Eagon-Northcott complexes generalize Koszul complexes and, if time permits, how we can apply them to study some examples in algebraic geometry, such as finite sets of points in the projective plane and rational normal curves.

Colloquium Series
Tuesday, November 08, 2022, 4:00pm-5:00pm
1360 East Hall
Ziquan Zhuang (Johns Hopkins University/IAS, Princeton)
Canonical metrics on Fano varieties

Finding canonical metrics, such as Kähler-Einstein metrics, on compact Kähler varieties has been an intense topic of research for decades. I'll present some works (joint with Harold Blum, Yuchen Liu and Chenyang Xu) in this direction when the first Chern classes of the varieties are positive (such varieties are called Fano varieties). I'll focus on two particular aspects: the Yau-Tian-Donaldson conjecture, which says that the existence of Kähler-Einstein metrics on Fano varieties is equivalent to an algebraic stability condition called K-polystability, and an algebro-geometric analog of the Kähler-Ricci flow.

Student Analysis
Tuesday, November 08, 2022, 5:00pm-6:00pm
3096 East Hall
Ethan Zell (University of Michigan)
Mean Field Equilibria

We take a first look at a problem in mathematical game theory, formulated as an optimal control problem in continuous time. In this context, we introduce the foundational notions of Nash equilibrium and Hamilton-Jacobi-Bellman equations. Then, when considering large-player systems, we pose the abstraction of sending the number of players to infinity and formulate the so-called mean field game that arises. Finally, we mention the notion of equilibrium in the mean-field game and demonstrate how it naturally corresponds to the finite-player Nash equilibrium. This talk is based on joint work with Asaf Cohen.
Learning Seminar in Algebraic Combinatorics  
Wednesday, November 09, 2022, 2:30pm-4:00pm  
4088 East Hall  
Nir Gadish (University of Michigan)  
Producing a Poisson cluster variety using dimer models

In the coming two talks we will discuss a construction of a cluster variety with a Poisson structure, coming from bipartite graphs on the torus. This week will start with some definitions and motivation for Poisson varieties, and proceed to a construction of a cluster variety from a (Newton) polygon in the plane. We will see that every bipartite graph has a natural algebraic torus attached - the moduli space of line bundles on the graph. These tori glue together along mutations of the graph, giving a cluster variety.
It is well established that the appearance and properties of self-assembled structures are affected by the geometry of their constituents. This is especially true for hard polyhedrally shaped particles, which interact solely via excluded volume to form a plethora of entropically stabilized crystal structures. Yet, a priori prediction of these structures is non-trivial for anything but the simplest of space-filling shapes, such as cubes, especially when the thermodynamically preferred structure differs from the densest packing structure. By sufficiently curving space, however, we can eliminate the geometric constraints that prevent polyhedra from forming locally dense packings and theoretically create tessellations for all regular polyhedra. Using Monte Carlo simulations, we show that most hard polyhedra belonging to the family of Platonic solids can self-assemble into space-filling crystal structures when constrained to the surface of a hypersphere. By increasing the hypersphere radius to gradually flatten space, we introduce geometric frustration that prevents the particles from tessellating the hypersphere, and inevitably introduces defects. Lastly, we compare systems assembled in curved and flat space by applying different local environment metrics and show that all the observed assemblies of Platonic shapes in Euclidean space can be interpreted as shadows of tessellations and defects on the hypersphere.

Talk in person in East Hall 4448 and on Zoom:

Join Zoom Meeting:
https://umich.zoom.us/j/94775967057

Meeting ID: 947 7596 7057

Sponsored by the Van Loo Symposium Fund
Financial/Actuarial Mathematics  
Wednesday, November 09, 2022, 3:00pm-4:00pm  
296 Weiser Hall  
Ibrahim Ekren (Florida State University)  
*Monge-Kantorovich Duality, Informed Trading, and Risk Aversion*  

We establish a novel connection between optimal transport theory and the dynamic version of the Kyle model of informed trading. Our methodology based on the Monge-Kantorovich duality and backward stochastic partial differential equations allows us to obtain the existence of equilibrium in all risk-neutral versions of the model studied in the literature and extend the existence results to novel cases. With risk-averse market makers, we show that liquidity is lower, assets exhibit short-term reversals, and risk premia depends on market maker inventories, which are mean reverting. We illustrate the model by showing that implied volatilities predict stock returns when there is informed trading in stocks and options and market makers are risk averse. Based on joint work with K. Back, S. Bose, R. Chhaibi, F. Cocquemas, A. Lioui, E. Noh, and L. Vy.

Student Arithmetic  
Wednesday, November 09, 2022, 3:00pm-4:00pm  
1866 East Hall  
Simran Khunger ()  
*Vinogradov Mean Value Theorem*  

I'll be discussing the Vinogradov Mean Value Theorem, which is a powerful tool for solving Waring's problem on how to write a fixed integer \( N \) as the sum of \( s \) \( k \)th powers, or finding zero free regions of the Riemann zeta function. We'll mostly be looking towards the Waring's problem perspective and will explore Wooley's technique of efficient congruencing. We'll also talk about Bourgain, Demeter, and Guth's 2015 resolution of VMVT via \( l^2 \) decoupling, a powerful technique in harmonic analysis.

Algebraic Geometry  
Wednesday, November 09, 2022, 4:00pm-5:20pm  
4096 East Hall  
Ziquan Zhuang (Johns Hopkins University)  
*Stable degenerations of klt singularities*  

Several years ago, Chi Li introduced the normalized volumes of valuations in his work on K-stability. The stable degeneration conjecture, due to Li and Xu, predicts that every klt singularity has a canonical "stable degeneration" induced by the minimizers of the normalized volume functions. I'll talk about the recent solution of this conjecture, focusing on its connection to certain finite generation property of valuations. Based on joint work with Chenyang Xu.
Suppose we run a random walk on a group. The corresponding Green metric assigns distances between group elements as follows: the distance between $x$ and $y$ is the negative logarithm of the probability that, when we start our random walk at $x$, we reach $y$. In this talk we'll discuss how to obtain orbital counting results for Green metrics by using techniques from ergodic theory.

The Solecki dichotomy in descriptive set theory, roughly stated, says that every Borel function on the real numbers is either a countable union of partial continuous functions or at least as complicated as the Turing jump. The Posner-Robinson theorem in computability theory, again roughly stated, says that every non-computable real looks like $0'$ relative to some oracle. Superficially, these theorems look similar: both roughly say that some object is either simple or as complicated as the jump. However, it is not immediately apparent whether this similarity is more than superficial. If nothing else, the Solecki dichotomy is about third order objects—functions on the real numbers—while the Posner-Robinson theorem is about second order objects—individual real numbers. We will show that there is a genuine mathematical connection between the two theorems by showing that the Posner-Robinson theorem plus determinacy can be used to give a short proof of a slightly weakened version of the Solecki dichotomy, and vice-versa.

Perinormal and globally perinormal domains were first introduced by Epstein and Shapiro in 2016. An integral domain $R$ is globally perinormal (resp. perinormal) if every going-down overring (resp. every local going-down overring) is a localization of $R$. I show that global perinormality is preserved in a pullback construction which encompasses a classical $D+M$ construction. In doing so, a result is given for the transfer of the property that every flat overring is a localization. I will also introduce notions of graded perinormality and graded global perinormality and give some results for descent of properties between a graded domain and its $0$th graded component.
Spectra are among the most fundamental objects in algebraic topology and appear naturally in the study of generalized cohomology theories, higher K-groups and cobordism invariants. My goal is to explain the modern perspective that spectra are natural homotopical analogues of abelian groups in a theory of "higher algebra," where one has algebraic structures like rings, modules, and tensor products internal to spectra.

This perspective promotes new interactions with other areas of mathematics. On the one hand, the existence of additional "chromatic primes" in higher algebra (interpolating between characteristic 0 and characteristic p) has shed light on mod p phenomena in geometry, number theory, and representation theory. On the other hand, the extension of algebraic ideas to higher algebra has been fruitful for topology: I will discuss work, joint with Robert Burklund and Tomer Schlank, which proves a higher algebra analogue of Hilbert's Nullstellensatz. In addition to initiating the study of "chromatic algebraic geometry," this work resolves a form of Rognes' "chromatic redshift" conjecture in algebraic K-theory.

The study of static vacuum Riemannian metrics arises naturally in differential geometry and general relativity. It plays an important role in scalar curvature deformation, as well as constructing vacuum Einstein spacetimes. Existence of static vacuum Riemannian metrics with prescribed Bartnik data - the induced metric and mean curvature of the boundary - is one of the most interesting problems in Riemannian geometry related to general relativity. It is also a problem on the global solvability of a natural geometric system of partial differential equations. In this talk I will present some recent progress towards the existence problem of static vacuum metrics based on joint works with Lan-Hsuan Huang.

http://www.math.lsa.umich.edu/seminars_events/ - Page 10/13
Thurston’s topological characterization of rational functions is a major theorem that characterizes when a certain type of topological mapping is equivalent to a rational function. We will make rigorous these notions and introduce Thurston’s theorem. We will then give an overview of applications such as the twisted rabbit problem and spiders.

Swimming and flying animals rely on the fluid around them to provide lift or thrust forces, leaving behind a distinct vortex wake in the fluid. The structure and size of the vortex wake is a blueprint of the animal’s kinematic trajectory, holding information about the forces and also the size, speed and direction of motion. This talk will introduce a bio-inspired oscillating turbine, which can be operated to generate energy from moving water through lift generation, in the same manner as flapping birds or bats. This style of turbines offers distinct benefits compared with traditional rotation-based turbines such as the ability to dynamically shift its kinematics for changing flow conditions, thus altering its wake pattern. Current efforts lie in predicting the vortex formation and dynamics of the highly structured wake such that it can be utilized towards cooperative motion within arrays of oscillating foils. Using numerical simulations, this talk will discuss efforts towards linking the fluid dynamic wake signature to the underlying foil kinematics, and investigating how that effects the energy harvesting performance of downstream foils. Two machine learning methodologies are introduced to classify, cluster and identify complex vorticity patterns and modes of energy harvesting, and inform more detailed modeling of arrays of oscillating foils.
A linear code is a subspace of $\mathbb{F}_q^n$ consisting of vectors that are pairwise distinct from each other at (hopefully) many different coordinates. One way of producing good codes comes from taking sections of line bundles on curves over finite fields. In this talk I will explain the construction, and continue on to discuss constructions and bounds regarding the number of rational points on a curve over a finite field, because, as we will see, curves with a large number of points relative to their genus produce the best codes.

We deal always with $r$-regular bipartite graphs or lattices. We state two results discussed in the talk. First, for the lattice gas of dimers on the hyper-rectangular lattice of any dimension, the first 20 terms in the virial expansion are positive. Second, for a graph of $v$ vertices we define a function $d(i)$ of the number of $i$-matchings by $\ln( N/r^i) - \ln(N'/v-1)^i)$, where $N$ is the number of $i$-matchings of the graph, $N'$ the number of $i$-matchings of its completion. We define delta by $\delta f(z) = f(z+1) - f(z)$. Then, if $j+k < 30$, the fraction of graphs with $v$ vertices that satisfy $\delta^k f(j) > 0$ approaches 1 as $v$ goes to infinity.

Geometric representability of 1-cycles on rationally connected threefolds

https://arxiv.org/abs/2208.12557
Special Events
Friday, November 11, 2022, 4:00pm-5:30pm
3096 East Hall
Jinho Baik & Asaf Cohen (UM)
Faculty Spotlight on Probability

During 2020-2021 I organized (in collaboration with GSAC) a series of “Faculty Spotlight” events, where faculty members gave brief introductions to their research, followed by questions, such as inquiries about advising style. After a hiatus, we will resume this activity.

The whole event should take up to 90 minutes.

The event is primarily intended for pre-candidates, but everyone is of course welcome. Following presentations by profs Baik and Cohen, there will be time for questions and a discussion.

I encourage all students without an advisor to go, even those who are currently not leaning towards working on the exact area being discussed. You may get ideas about how to think about the advisor search even if it will eventually be in a different area.

Variational Analysis and Optimization
Friday, November 11, 2022, 9:00am-10:00am
Virtual
Marco A. López (University of Alicante, Spain)
A new tour on the subdifferential of the supremum function

This talk is a kind of survey presenting various characterizations of the subdifferential of the pointwise supremum of an arbitrary family of convex functions, as well as some featured applications. Starting by the maximum generality framework, we move after to particular contexts in which some continuity and compacity assumptions are either imposed or enforced via processes of compactification of the index set and regularization of the data functions. Some relevant applications of the general results are presented, in particular to derive rules for the subdifferential of the sum, and for convexifying a general (unconstrained) optimization problem. The last part deals with some specific constraint qualifications for the convex optimization problem with an arbitrary set of constraints, and also contains different sets of KKT-type optimality conditions appealing to the subdifferential of the supremum function.