(1) Let $c$ be a real number such that $c^4 = 5$. Determine (with proof) which of these extensions are normal: $\mathbb{Q}(ic^2)/\mathbb{Q}$, $\mathbb{Q}(c+ic)/\mathbb{Q}(ic^2)$, $\mathbb{Q}(c+ic)/\mathbb{Q}$.

(2) Give a nice description of the splitting field of each of the following polynomials over $\mathbb{Q}$, and in particular determine the degree of the splitting field (as a field extension of $\mathbb{Q}$): $x^2 - 2$, $x^2 - 1$, $x^3 - 2$, $(x^3 - 2)(x^2 - 2)$, $x^2 + x + 1$, $x^6 + x^3 + 1$, $x^5 - 7$.

(3) Problem 4.1 in chapter 16 of Artin.

(4) A field $K$ is called perfect if every finite extension $L/K$ is separable. Show that $K$ is perfect if and only if one of these holds:
   (1) $K$ has characteristic 0, or
   (2) $K$ has characteristic $p$ with $p > 0$, and also every element of $K$ has a $p$-th root in $K$.

(5) Let $p$ be prime, let $L := \mathbb{F}_p(X, Y)$ be the field of rational functions in two variables, and put $K := \mathbb{F}_p(X^p, Y^p)$. Show that $L/K$ is a finite-degree extension, but that $L \neq K(z)$ for any $z \in L$. Also exhibit infinitely many distinct fields $F$ such that $K \subset F \subset L$ (don’t just cite problem 6 for this, instead you should name the fields $F$ here).

(6) Let $L/K$ be a finite-degree field extension, where $K$ is infinite. Show that $L$ can be written as $K(\alpha)$ for some $\alpha \in L$ if and only if there exist only finitely many fields $F$ with $K \subset F \subset L$.
   (I will post hints for this on piazza.)