(1) Let $R$ be a Dedekind domain with field of fractions $K$, and let $S$ be a finite set of nonzero prime ideals of $R$.
(a) Show that $R^\times$ is the intersection of $R_p^\times$, taken over all maximal ideals $p$ of $R$.
(b) Show that there is a canonical exact sequence of abelian groups
\[ 1 \rightarrow R^\times \rightarrow (R^S)^\times \rightarrow \bigoplus_{P \in S} (K^\times / R_P^\times) \rightarrow \text{Cl}(R) \rightarrow \text{Cl}(R^S) \rightarrow 1. \]
(c) Show that $K^\times / R_P^\times \cong \mathbb{Z}$ for each $P \in S$.
(d) Show that if $K$ is a number field and $R = \mathcal{O}_K$ then $(R^S)^\times \cong W_K \times \mathbb{Z}^{r_1 + r_2 - 1 + |S|}$, where $W_K$ is the group of roots of unity in $K$, $r_1$ is the number of real embeddings of $K$, and $r_2$ is the number of complex-conjugate pairs of non-real complex embeddings of $K$.

(2) Let $a, b$ be squarefree integers congruent to 1 mod 3 such that $a, b, 1$ are pairwise distinct, and let $K = \mathbb{Q}(\sqrt{a}, \sqrt{b})$. Show that it is not possible to write $\mathcal{O}_K = \mathbb{Z}[\alpha]$ with $\alpha \in K$.
(You may assume that if $L, M, N$ are number fields with $L \subseteq M \cap N$, and $p$ is a prime ideal of $\mathcal{O}_L$ which splits completely in both $\mathcal{O}_M$ and $\mathcal{O}_N$, then $p$ splits completely in $\mathcal{O}_{LM}$. This fact will be proved in class next week.)