For each prime \( p > 2 \), show that \( 16 = c^8 \) for some \( c \in \mathbb{Q}_p \).

(2) Show that both \((x^2 - 2)(x^2 - 17)(x^2 - 34)\) and \((x^3 - 37)(x^2 + 3)\) have roots in \( \mathbb{Q}_p \) for every \( p \).

Massive extra credit: show the same thing for \( 3x^4 + 4y^4 - 19z^4 = 0 \).

(3) Let \( K \) be a field which is complete with respect to a non-archimedean absolute value \(|\cdot|\), let \( a_0, a_1, \ldots \) be a sequence of elements of \( K \), and define

\[
R := \limsup_n a_n |1/n|
\]

which is an element of \([0, +\infty)\). Show that \( D := \{ x \in K : \sum_{n=0}^{\infty} a_n x^n \text{ converges} \} \) satisfies:

1. If \( R = 0 \) then \( D = \{ 0 \} \).
2. If \( R = \infty \) then \( D = K \).
3. If \( 0 < R < \infty \) and \( \lim_n |a_n| R^n = 0 \) then \( D = \{ x \in K : |x| \leq R \} \).
4. If \( 0 < R < \infty \) and \( |a_n| R^n \nrightarrow 0 \) then \( D = \{ x \in K : |x| < R \} \).

(4) If \( p \) is an odd prime, \( t \in \mathbb{Z}_p \), and \( x \in p\mathbb{Z}_p \), show that the binomial series

\[
G(t, x) := \sum_{n=0}^{\infty} \binom{t}{n} x^n
\]

converges. If \( t = u/v \) with \( u, v \in \mathbb{Z} \) and \( v > 0 \) and \( p \nmid v \), then show that \( G\left(\frac{u}{v}, x\right)^v = (1 + x)^u \). Show in particular that if \( p = 7 \), \( t = 1/2 \) and \( x = 7/9 \) then the series converges to \( 4/3 \) in \( \mathbb{R} \) and to a 7-adic number \( \alpha \neq 4/3 \) in \( \mathbb{Q}_7 \).

(5) (a) Determine the set of elements in \( \mathbb{Q}_p \) for which the power series

\[
\log_p(x) := \sum_{n=1}^{\infty} (-1)^{n+1} \frac{(x - 1)^n}{n}
\]

converges.

(b) Determine the set of elements in \( \mathbb{Q}_p \) for which the power series

\[
\exp_p(x) := \sum_{n=0}^{\infty} \frac{x^n}{n!}
\]

converges.

(c) If \( a \in \mathbb{Q}_p \) is small enough, show that \( \exp_p(\log_p(a)) = a \). How close is “close enough”?