(1) For each prime $p > 2$, show that $16 = c^8$ for some $c \in \mathbb{Q}_p$.

(2) Show that both $(x^2 - 2)(x^2 - 17)(x^2 - 34)$ and $(x^3 - 37)(x^2 + 3)$ have roots in $\mathbb{Q}_p$ for every $p$, but have no roots in $\mathbb{Q}$.

**Massive extra credit:** Show that for each prime $p$ the equation $3x^4 + 4y^4 - 19z^4 = 0$ has solutions with $x, y, z \in \mathbb{Q}_p$ which are not all zero, but no such solutions with $x, y, z \in \mathbb{Q}$.

(3) Let $K$ be a field which is complete with respect to a non-archimedean absolute value $|\cdot|$, let $a_0, a_1, \ldots$ be a sequence of elements of $K$, and define
\[ R := \limsup_n |a_n|^{1/n} \]
which is an element of $[0, +\infty]$. Show that $D := \{x \in K : \sum_{n=0}^\infty a_n x^n \text{ converges}\}$ satisfies:

1. If $R = 0$ then $D = \{0\}$.
2. If $R = \infty$ then $D = K$.
3. If $0 < R < \infty$ and $\lim_n |a_n| R^n = 0$ then $D = \{x \in K : |x| \leq R\}$.
4. If $0 < R < \infty$ and $|a_n| R^n \not\to 0$ then $D = \{x \in K : |x| < R\}$.

(4) If $p$ is an odd prime, $t \in \mathbb{Z}_p$, and $x \in p\mathbb{Z}_p$, show that the binomial series
\[ G(t, x) := \sum_{n=0}^\infty \binom{t}{n} x^n \]
converges. If $t = u/v$ with $u, v \in \mathbb{Z}$ and $v > 0$ and $p \nmid v$, then show that $G(u/v, x)^v = (1 + x)^u$. Show in particular that if $p = 7$, $t = 1/2$ and $x = 7/9$ then the series converges to $4/3$ in $\mathbb{R}$ and to a 7-adic number $\alpha \neq 4/3$ in $\mathbb{Q}_7$.

(5) (a) Determine the set of elements in $\mathbb{Q}_p$ for which the power series
\[ \log_p(x) := \sum_{n=1}^\infty (-1)^{n+1} \frac{(x - 1)^n}{n} \]
converges.
(b) Determine the set of elements in $\mathbb{Q}_p$ for which the power series
\[ \exp_p(x) := \sum_{n=0}^\infty \frac{x^n}{n!} \]
converges.
(c) If $a \in \mathbb{Q}_p$ is small enough, show that $\exp_p(\log_p(a)) = a$. How close is “close enough”?