Inertial support of distinguished representations

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Distinguished representations

Let H be a subgroup of a group G and let π be a representation of G. We say that π is *H-distinguished* if there exists a nonzero H-invariant linear functional on the space of π.

In this talk, all representation spaces are complex vector spaces.

We assume that G is a connected reductive *p*-adic group: $G = \mathbf{G}(F)$, where $\mathbf{G}$ is a connected reductive F-group and F is a local nonarchimedean field. (For technical reasons, we assume that the residual characteristic of F is odd.) When studying K-types contained in distinguished representations of G, we will be working with distinction of smooth representations of profinite groups.

We say that θ is an *involution* of *G* if θ is an *F*-automorphism of **G** of order two. Let *H* be the group of fixed points of θ. We are interested in understanding (parametrizing, whenever possible) the *H*-distinguished irreducible admissible representations of *G*. These are the representations which play a role in harmonic analysis on the *p*-adic symmetric variety G/H.

In some situations, we consider a slight generalization of the notion of distinction: If χ is a quasicharacter of *H*, let

$$\mathrm{Hom}_H(\pi, \chi) = \{\, \lambda \in V^* \mid \lambda \circ \pi(h) = \chi(h)\lambda \,\forall\, h \in H \,\}.$$

If $\mathrm{Hom}_H(\pi, \chi)$ is nonzero, we say that π is (H, χ)-*distinguished*.

Various examples of *H*-distinguished representations of *G* occur as irreducible subquotients of representations of the form $\text{Ind}_P^G \tau$, where

- *M* is a θ-stable Levi factor of a (not necessarily θ-stable) parabolic subgroup *P* of *G*.
- τ is an irreducible supercuspidal representation of *M* such that some unramified twist of τ is M^θ-distinguished.

In a recent paper(J. No. Theory, 2014), we obtain information about distinction of types contained in the (inertial) supports of distinguished depth-zero irreducible smooth representations.

Theorem

The support of a depth-zero irreducible smooth H-distinguished representation of G contains a pair (M, τ) where M is a θ-stable Levi subgroup of G and τ is an irreducible (depth-zero) supercuspidal representation of M containing a (depth-zero) unrefined minimal K-type (K_M, ρ_M) such that K_M is θ-stable and ρ_M is K_M^θ-distinguished.

This suggests that the inertial supports of distinguished irreducible smooth representations may contain distinguished representations of θ-stable Levi subgroups. We study the properties of K-types contained in distinguished "tame" representations and their inertial supports and use these properties to show that the inertial supports do have this property.

Suppose that (K, ρ) is a K-type contained in an irreducible smooth distinguished representation π. Then, because π is distinguished, there exists $g \in G$ such that $\mathrm{Hom}_{K \cap {}^g H}(\rho, 1) \neq 0$. When (K, ρ) satisfies:

- (K, ρ) is a G-cover of a "sufficiently large" type contained in the inertial support of π

- The inertial support of π is "tame"

- Certain hypotheses concerning quasicharacters are satisfied,

then we can show that the inertial support of π is "distinguished". More precise statements will be made later.

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Let τ and τ' be irreducible supercuspidal representations of
Levi subgroups M and M' of G, respectively. The pairs
(M, τ) and (M', τ') are said to be *inertially equivalent* if
there exist $g \in G$ and $\chi \in X(M')$ such that ${}^g M = M'$ and
${}^g \tau \simeq \tau' \chi$. The inertial equivalence class of a pair (M, τ) will
be denoted by $[M, \tau]_G$.

Recall that if π is an irreducible smooth representation of G,
there exists a pair (M, τ), which is unique up to conjugacy,
consising of a Levi subgroup M of G and an (equivalence
class of an) irreducible supercuspidal representation τ of M
such that for any parabolic subgroup $P \in \mathcal{P}(M)$, π occurs as
an subquotient of $\mathrm{Ind}_P^G \tau$. (Here, $\mathcal{P}(M)$ is the set of parabolic
subgroups of G having Levi factor M.) The conjugacy class
of the pair (M, τ) is called the *(cuspidal) support* of π. The
inertial equivalence class $\mathfrak{I}(\pi) := [M, \tau]_G$ is called the
inertial support of π.

We will say that an inertial equivalence class of *G* is
θ-*distinguished* (or just *distinguished*) if it contains a pair
(M, τ) such that $\theta(M) = M$ and $\mathrm{Hom}_{M^\theta}(\tau, 1) \neq 0$.

Remark

The group *G* acts on the set of involutions of *G*. If $g \in G$,
the involution $g \cdot \theta$ is defined by

$$(g \cdot \theta)(x) = g \, \theta(g^{-1} x g) g^{-1}, \qquad x \in G.$$

Note that $G^{g \cdot \theta} = g G^\theta g^{-1}$. It is clear that an inertial
equivalence class is θ-distinguished if and only if it is
$g \cdot \theta$-distinguished for every $g \in G$.

Question

*Let π be an irreducible smooth H-distinguished
representation of G. Is the inertial support $\mathfrak{I}(\pi)$ of π
θ-distinguished?*

Example

Let G be a split group and let π be an (irreducible) unramified representation of G. As shown by Helminck and Wang, there exists a θ-stable maximal F-split torus A in G. The pair $(A, 1)$ belongs to the inertial support $\mathfrak{I}(\pi)$ of π. Hence $\mathfrak{I}(\pi)$ is θ-distinguished (even when π is not distinguished).

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Example:

- $G = \mathbf{GL}_{2n}(F), \quad H = G^\theta = \mathbf{Sp}_{2n}(F)$,
- $M = \mathbf{GL}_n(F) \times \mathbf{GL}_n(F)$ such that
  $\theta(g_1, g_2) = ({}^t g_2^{-1}, {}^t g_1^{-1}), g_j \in \mathbf{GL}_n(F)$.

Note that $M^\theta = \{ (g, {}^t g^{-1}) \mid g \in \mathbf{GL}_n(F) \}$. An irreducible
smooth representation $\tau_1 \otimes \tau_2$ of M is M^θ-distinguished if
and only if $\tau_2 \simeq \tau_1$.

(Fix an irreducible smooth representation τ' of $\mathbf{GL}_n(F)$. Let
$\mathcal{A}$ be a nonzero intertwining operator between the
representation $g \mapsto \tau'({}^t g^{-1})$ and $\widetilde{\tau}'$. Define a linear form λ
on $V_{\tau'} \otimes V_{\tau'}$ by $\lambda(v_1 \otimes v_2) = \langle \mathcal{A} v_2, v_1 \rangle$, $v_1, v_2 \in V_{\tau'}$. It is
easy to see that λ is M^θ-invariant.)

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Assume that τ' is supercuspidal. If P is a parabolic subgroup of G with Levi factor M, the representation $\mathrm{Ind}_P^G(\tau' \otimes \tau')$ is irreducible and hence has a Whittaker model (since τ' has a Whittaker model). According to a result of Heumos and Rallis, $\mathrm{Ind}_P^G(\tau' \otimes \tau')$ is not H-distinguished. So we cannot construct H-distinguished representations of G by inducing from M^θ-distinguished supercuspidal representations of M.

Let $\nu(g) = |\det g|_F^{-1}$, $g \in \mathbf{GL}_n(F)$ and $\tau_\nu = \tau' \otimes \nu\tau'$. For P a particular parabolic subgroup of G with Levi factor M, the unique irreducible quotient π_{τ_ν} of the reducible representation $\mathrm{Ind}_P^G \tau_\nu$ is H-distinguished. (This is due to Heumos and Rallis.) The pair (M, τ_ν) belongs to the support of π_{τ_ν} and τ_ν is not M^θ-distinguished. However, the representation $\tau' \otimes \tau'$ belongs to $\mathfrak{I}(\pi_{\tau_\nu})$ and $\tau' \otimes \tau'$ is M^θ-distinguished. In this example, none of the distinguished pairs in the inertial support $\mathfrak{I}(\pi_{\tau_\nu})$ belong to the support of π_{τ_ν}.

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In general, there can be several pairs in the (inertial) support of a distinguished irreducible admissible representation having the property that the associated Levi subgroups are θ-stable. It is possible that more than one such pair is θ-distinguished, but this is not necessarily the case.

Returning to the above example, suppose that n is even. Then there exists $g \in \mathbf{GL}_{2n}(F)$ such that $L := {}^g M$ is θ-stable and $L^\theta \simeq \mathbf{Sp}_n(F) \times \mathbf{Sp}_n(F)$. By a result of Heumos and Rallis, there are no $\mathbf{Sp}_n(F)$-distinguished supercuspidal representations of $\mathbf{GL}_n(F)$. Note that the pair $(L, {}^g \tau_\nu)$ belongs to the support of π_{τ_ν}. Hence the inertial support of π_{τ_ν} contains some pairs of the form (L, σ), but no such pair is θ-distinguished.

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Suppose that

- *G* splits over a tamely ramified extension of *F*.
- The residual characteristic p of F is not a torsion prime for $\Psi(G)^\vee$, the root datum dual to the root datum $\Psi(G)$ of $G \otimes_F \overline{F}$.

A *G*-datum (as defined by J.-L. Kim and J.-K. Yu) is a 5-tuple

$$((\vec{G}, M^0), (y, \iota), \vec{r}, (K_{M^0}, \rho_{M^0}), \vec{\phi})$$

satisfying the following conditions (**D1**–**D5**):

D1 $\vec{G} = (G^0, G^1, \ldots, G^d)$ is a tamely ramified twisted Levi sequence in *G*, and M^0 is a Levi subgroup of G^0.

D2 y is a point in $\mathcal{B}(M^0)$ and $\{\iota\}$ is a commutative diagram of $\vec{s}$-generic embeddings of buildings relative to y, where $\vec{s} = (0, r_0/2, \ldots, r_{d-1}/2)$.

D3 $\vec{r} = (r_0, \ldots, r_d)$ is a sequence of real numbers satisfying $0 < r_0 < r_1 < \cdots < r_{d-1} \leq r_d$ if $d > 0$ and $0 \leq r_0$ if $d = 0$.

D4 (K_{M^0}, ρ_{M^0}) is such that
$((G^0, M^0), (y, \iota : \mathcal{B}(M^0) \to \mathcal{B}(G^0)), (K_{M^0}, \rho_{M^0}))$ is depth
zero datum.

D5 $\vec{\phi} = (\phi_0, \phi_1, \ldots, \phi_d)$ is a sequence of quasicharacters,
where ϕ_i is a quasicharacter of G^i, and, if $d > 0$, ϕ_i is
G^{i+1}-generic of depth r_i relative to x for all $x \in \mathcal{B}(G^i)$
for $0 \leq i \leq d - 1$.

Notation: $\mathcal{B}(G)$ is the extended Bruhat-Tits building of G.

Notation needed for **D2** (more comments on next frame):

Let A_{M^0} be the F-split component of the centre of M^0. If
$d > 0$, for $1 \leq i \leq d$, let M^i be the centralizer of A_{M^0} in G^i.
Then M^i is a Levi subgroup of G^i and M^i is a twisted Levi
subgroup of M^{i+1} (for $i \leq d - 1$).

Comments about **D2**: We have embeddings

$$\iota : \mathcal{B}(M^i) \to \mathcal{B}(G^i),$$
$$\iota : \mathcal{B}(M^i) \to \mathcal{B}(M^{i+1}), \quad i \leq d - 1,$$
$$\iota : \mathcal{B}(G^i) \to \mathcal{B}(G^{i+1}), \quad i \leq d - 1.$$

These form a commutative diagram $\{\iota\}$ of embeddings. (We haven't figured out how to make a commutative diagram in latex.)

If s is a nonnegative real number, M is a Levi subgroup of G, and $y \in \mathcal{B}(M)$, then $\iota : \mathcal{B}(M) \to \mathcal{B}(G)$ is s-generic with respect to y if

$$U_\alpha \cap G_{\iota(y),s} = U_\alpha \cap G_{\iota(y),s^+} \text{ for } \alpha \in \Phi(G, S, F) \setminus \Phi(M, S, F).$$

Here, S is a maximal F-split torus of M such that y belongs to the apartment in $\mathcal{B}(M)$ associated to S and U_α is the root subgroup of G associated to α.

For $y \in \mathcal{B}(M^0)$ and $\vec{s} = (s_0, \ldots, s_{d-1})$, $\{\iota\}$ is $\vec{s}$-generic with respect to y if $\iota : \mathcal{B}(M^i) \to \mathcal{B}(G^i)$ is s_i-generic with respect to $\iota(y)$, $0 \leq i \leq d - 1$.

Let Σ be a *G*-datum. Set $M = M(\Sigma) = M^d$. (Recall that M^d is the centralizer in *G* of the *F*-split torus A_{M^0}.) Kim and Yu define a compact open subgroup $K = K_\Sigma$ of *G* and an irreducible smooth representation $\rho = \rho_\Sigma$ of K_Σ. Let $K_M = K \cap M$ and $\rho_M = \rho_M(\Sigma) = \rho \,|\, K_M$.

Theorem

(Kim and Yu) *With notation as above, (K_M, ρ_M) is a supercuspidal type on M and (K, ρ) is a G-cover of (K_M, ρ_M).*

When we say that (K_M, ρ_M) is a supercuspidal type on *M*, we mean that (K_M, ρ_M) is a type on *M* and every irreducible smooth representation of *M* that contains (K_M, ρ_M) is supercuspidal.

The requirement (see condition **D2**) that $\{\iota\}$ be $\vec{s}$-generic is essential for (K, ρ) to be a *G*-cover of (K_M, ρ_M).

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We say that a *G*-datum

$$\Sigma = ((\vec{G}, M^0), (y, \iota), \vec{r}, (K_{M^0}, \rho_{M^0}), \vec{\phi})$$

is θ-*symmetric* if

S1 $\theta(G^i) = G^i$, $0 \leq i \leq d$

S2 $\phi_i \circ \theta = \phi_i^{-1}$, $0 \leq i \leq d$

S3 $\theta(M_y^0) = M_y^0$

Remark

The subgroup M_y^0 is a maximal parahoric subgroup of M^0 (because condition **D4** guarantees that K_{M^0} contains M_y^0 and $\rho_{M^0} \mid M_y^0$ is a depth-zero supercuspidal type on M^0).)

Remark

When Σ is θ-symmetric, $\theta(M^i) = M^i$ for $0 \leq i \leq d$. In particular, $M = M(\Sigma)$ is a θ-stable Levi subgroup of G. Moreover, if K_{M^0} is chosen to be θ-stable, then K_M is θ-stable. However, K might not be θ-stable.

Here is an example where K is not θ-stable:

Example

$$G = GL_2(F), \quad \theta(g) = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} g \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},$$

χ a depth-zero quasicharacter of $F^\times$,

P the upper-triangular Borel subgroup.

The representation $\pi_\chi := \text{Ind}_P^G(\chi \otimes \chi^{-1})$ is G^θ-distinguished. A depth-zero minimal K-type (G_x, ρ) of π_χ has the property that G_x is an Iwahori subgroup of G. When χ^2 is nontrivial on $\mathfrak{o}_F^\times$, then $\theta(G_x) \neq G_x$ whenever $\text{Hom}_{G_x^\theta}(\rho, 1)$ is nonzero. Here, Σ is a depth-zero G-datum and $(K_\Sigma, \rho_\Sigma) = (G_x, \rho)$. (*Notation*: $\mathfrak{o}_F$ is the ring of integers of F.)

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Hypothesis C(**G**)

If ϕ is a quasicharacter of G of positive depth r and $x \in \mathcal{B}(G)$, then $\phi \,|\, G_{x,(r/2)^+}$ is realized by an element of $\mathfrak{z}^* \cap \mathfrak{g}^*_{x,-r}$. (Here, $\mathfrak{z}^*$ is the dual of the centre of the Lie algebra $\mathfrak{g}$ of G, and $\mathfrak{g}^*_{x,-r}$ is the Moy-Prasad filtration lattice of $\mathfrak{g}$ associated to x and $-r$.)

Theorem

Assume that Hypothesis C(G′) holds for all twisted Levi subgroups G′ of G. Let Σ be a G-datum, $K = K(\Sigma)$, $\rho = \rho(\Sigma)$, $M = M(\Sigma)$, $K_M = K \cap M$ and $\rho_M = \rho \,|\, K_M$. Suppose that $\mathrm{Hom}_{K^\theta}(\rho, 1) \neq 0$ (that is, there exist nonzero K^θ-stable vectors in the space of ρ). Then, after replacing Σ with G-datum $\dot{\Sigma}$ such that $K(\dot{\Sigma}) = K(\Sigma)$ and $\rho(\dot{\Sigma}) \simeq \rho(\Sigma)$, we may assume that Σ is θ-symmetric. In that case, $\mathrm{Hom}_{K^\theta}(\rho, 1) \simeq \mathrm{Hom}_{K_M^\theta}(\rho_M, 1)$.

Remark

A *G*-datum Σ is *cuspidal* if and only if $M(\Sigma) = G$. In this case, the type (K_Σ, ρ_Σ) is supercuspidal and the above result is proved in joint work with Jeff Hakim on distinction of tame supercuspidal representations.

Comments about the proof:

- Use methods from joint work with Jeff Hakim to show that Σ can be taken to be "weakly" θ-symmetric (satisfying **S1** and **S2**).

- Once **S1** and **S2** hold, we can reduce to the depth-zero setting. If Σ is not cuspidal, we can no longer use methods from joint work with Jeff Hakim. Instead, we apply results from the paper on depth-zero distinguished representations (results mentioned at the beginning of the talk) to show that we can arrange that **S3** holds as well.

- Now that we can assume Σ is θ-symmetric, we may use the properties of *G*-covers to see that

$$\mathrm{Hom}_{K^\theta}(\rho, 1) \simeq \mathrm{Hom}_{K_M^\theta}(\rho_M, 1).$$

If the group G splits over a tamely ramified extension of F, we say that an irreducible smooth representation of G is *tame* if the supercuspidal representations occurring in the inertial support $\mathfrak{I}(\pi)$ of π are among those constructed by J.-K. Yu. If Σ is a G-datum and π is an irreducible smooth representation of G containing the type (K_Σ, ρ_Σ), then π is tame.

Our results hold for depth-zero irreducible smooth representations and for those tame representations containing types associated to G-data. If p is sufficiently large, this is all tame representations of G.

If Σ is a *G*-datum, then, because (K_Σ, ρ_Σ) is a *G*-cover of (K_M, ρ_M), the Bushnell-Kutzko theory of types says that (K_Σ, ρ_Σ) is a type on *G*. Moreover, there is a finite collection $\mathfrak{S}(\Sigma)$ of inertial equivalence classes on *G* such that

$$\mathfrak{R}^{\mathfrak{S}(\Sigma)}(G) = \prod_{\mathfrak{s} \in \mathfrak{S}(\Sigma)} \mathfrak{R}^{\mathfrak{s}}(G) = \mathfrak{R}_{\rho_\Sigma}(G).$$

Here, a smooth representation (π, V) of *G* belongs to the subcategory $\mathfrak{R}^{\mathfrak{s}}(G)$ of $\mathfrak{R}(G)$ (the category of smooth representations of *G*) if and only if every irreducible subquotient of π has inertial support $\mathfrak{s}$.

The objects of the subcategory $\mathfrak{R}_{\rho_\Sigma}(G)$ are the smooth representations (π, V) having the property that V is generated by its ρ_Σ-isotypic subspace.

Similarly, there is a finite set $\mathfrak{S}_M(\Sigma)$ of supercuspidal inertial equivalence classes on M determined by the supercuspidal type (K_M, ρ_M). In fact, $\mathfrak{S}(\Sigma)$ is determined by $\mathfrak{S}_M(\Sigma)$. We haven't explained it here, but we can arrange to choose K_M so that $\mathfrak{S}^M(\Sigma) = \{[M, \tau]_M\}$ where τ is a tame supercuspidal representation of M.

Remark
Bushnell and Kutzko showed that this happens when $\tau = $ c-$\mathrm{Ind}_J^M \kappa$, J is open compact-mod-centre subgroup of M, K_M is the unique maximal compact open subgroup in J and ρ_M is an irreducible constituent of $\kappa \,|\, K_M$.)

When $\mathfrak{S}^M(\Sigma) = \{[M, \tau]_M\}$, then $\mathfrak{S}(\Sigma) = \{[M, \tau]_G\}$. When this happens, we say that the *G*-datum Σ is *maximal*.

Remark

When Σ is maximal, K_{M^0} is the maximal compact open subgroup in the normalizer of M_y^0 in M^0. That is, the depth-zero G^0-datum

$$((G^0, M^0), (y, \iota : \mathcal{B}(M^0) \to \mathcal{B}(G^0)), (K_{M^0}, \rho_{M^0}))$$

is maximal.

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We emphasize that our results are valid for irreducible depth-zero representations even when *G* does not split over a tamely ramified extension.

Theorem

Let (π, V) be an H-distinguished irreducible smooth representation of G. Assume that

1. *If the depth of π is positive, π contains (K_Σ, ρ_Σ), where Σ is a maximal G-datum.*

2. *Hypothesis C(G') holds for all twisted Levi subgroups G' of G.*

Then $\Im(\pi)$ is θ-distinguished.

Remark

If $G = \mathbf{GL}_n$ or more generally $G = R_{E/F}\mathbf{GL}_n$, where E/F is tamely ramified finite extension, in order for the theorem to hold, it suffices to assume that *p* is odd.

Let π be an *H*-distinguished admissible representation of *G*. If $\lambda \in \mathrm{Hom}_H(\pi, 1)$ and $v \in V$, the function $g \mapsto \lambda(\pi(g)v)$ might not be a matrix coefficient of π (because λ might not be smooth). Such a function is called a *relative matrix coefficient* (or *generalized matrix coefficient*) of π.

We say that π is *(H-)relatively supercuspidal* if all of the relative matrix coefficients of π are compactly supported modulo *HZ*. (Here, *Z* is the centre of *G*.) (This notion was originally defined by Kato and Takano, and Lagier (independently).)

Kato and Takano proved a symmetric space analogue of the Jacquet Subrepresentation Theorem (stated on the next frame). A parabolic subgroup *P* of *G* is θ-split if $P \cap \theta(P)$ is a Levi factor of *P*.

Theorem

(Kato and Takano) *If an H-distinguished irreducible admissible representation π of G is not relatively supercuspidal, then there exist a proper θ-split parabolic subgroup P of G and an irreducible M^θ-relatively supercuspidal representation τ of $M := P \cap \theta(P)$ such that π is a subrepresentation of $\operatorname{Ind}_P^G \tau$.*

It is known that all *H*-distinguished irreducible supercuspidal representations of *G* are *H*-relatively supercuspidal.

However, there exist pairs (*G*, *H*) having the property that there are no *H*-distinguished supercuspidal representations of *G*. The pair $(\mathbf{GL}_{2n}(F), \mathbf{Sp}_{2n}(F))$ is such an example.

In certain settings, we have necessary and sufficient
conditions for existence of distinguished tame supercuspidal
representations.

An element *g* of *G* is *θ-split* if $\theta(g) = g^{-1}$.

Theorem

Let $\mathbf{G} = R_{E/F}\mathbf{GL}_n$ *where* E/F *is tamely ramified. There
exist distinguished tame supercuspidal representations of*
G^θ *if and only if there exist G-regular elliptic* θ*-split tamely
ramified elements in G.*

In this setting, if *p* is greater than *n* and there are no θ-split

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The **Sp**$_{2n}(F)$-distinguished (nonsupercuspidal) representation π_{τ_ν} of **GL**$_{2n}(F)$ from the earlier example is a relatively supercuspidal representation.

Using our results about distinction of types contained in inertial supports of distinguished tame representations, we have developed methods of constructing families of relatively supercuspidal representations. Although we obtain many relatively supercuspidal representations, the construction would have to be extended further in order to exhaust all relatively supercuspidal representations with tame support.