

**Undergraduate Math Club
Fall 2005
2nd floor Nesbitt Common Room
Sept. 22, 4:10-5:00pm
(free pizza and pop, as always)**

The Fifteen Theorem

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Abstract

A remarkable theorem of Bhargava, Conway, and Schneeberger states that any positive-definite quadratic form with integer matrix (all terms to be explained in the talk!) represents all natural numbers if it represents those up to 15. In particular, since the first 15 natural numbers are sums of four squares, so are all natural numbers! As another example, Ramanujan proved that every natural number can be written as a square plus twice a square plus three times a square plus five times a square; by the Fifteen Theorem, to prove Ramanujan's result we simply need to check it for the natural numbers up to 15. Perhaps the most astonishing fact about this theorem is that, in an area going back 300 years, it was discovered only about 5 years ago.

I will explain the precise statement of the theorem, give examples, and give some ideas in the proof. I will not assume familiarity with quadratic forms or last week's talk on the four-square theorem.

