# Undergraduate Math Club
# Fall 2005
# 2nd floor Nesbitt Common Room
# Sept. 29, 4:10-5:00pm
# (free pizza and pop, as always)

# Cross-ratio and conics

# Professor Brian Conrad

## Abstract

A conic in the plane is a smooth curve defined by a degree-2 polynomial in two variables. The Greeks did not have coordinate geometry, so they defined a conic to be the intersection of a cone with a plane not passing through the vertex; this gave rise to three classes of conics, namely the ellipses, hyperbolas, and parabolas. In this way, the Greeks used 3-dimensional space to provide a uniform geometric definition for conics in the plane. The intervention of space outside of the plane made their study of conics very cumbersome.

It is natural to ask for a uniform geometric definition of a conic that does not require leaving the plane. We introduce (in two ways!) the geometric notion of cross-ratio that beautifully answers this question, and we will see an interesting application to the geometry of conics.