Kakeya's problem: rotating needles in the plane

Marie Snipes

Abstract

Given a needle lying on a table, what is the smallest area in which you can turn the needle around without lifting it off the table? Clearly, you can just rotate the needle around its center of mass, but can you do better? It was conjectured by Kakeya in the early 20th century that a 3-pointed deltoid (which is not convex) achieves the minimal possible area, but to everyone's astonishment Besicovitch showed a few years later that this was spectacularly wrong: if your table is large enough, you can turn the needle around in a set of arbitrarily small area! Not only is this a surprising and fun fact to prove, but perhaps more remarkably it has many interesting applications to a diverse array of problems in mathematics, some of which will be discussed.