Michigan Math Club

Thursday at 4pm in the Nesbitt Room

Free Pizza and Pop

The Fáry-Milnor Theorem

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Abstract for 08 November 2018

Let $t \mapsto X(t)$ be a smooth space curve, and let $\mathbf{T}(t)$ be its unit tangent vector. Consider the curve on the unit sphere traced-out by $\mathbf{T}(t)$; the length of this curve is called the *total curvature*. In 1929, Fenchel proved that if $X(t)$ is a closed curve, then its total curvature is greater than or equal to 2π. The famous Fáry-Milnor Theorem from 1950 says that if the simple closed curve $X(t)$ is knotted, then the total curvature is greater than 4π.

We will discuss this theorem and Milnor's proof of it, which relies on the following (obvious!) result: Let C be a nice curve on the unit sphere, and for each great circle Γ, let $n(\Gamma)$ be the number of points in $C \cap \Gamma$. Then the length of C is equal to π times the average value of $\Gamma \mapsto n(\Gamma)$.