Undergraduate Math Club Winter 2005 2nd floor Nesbitt Common Room January 20, 4:05-5:00pm (free pizza and pop, as always)

Counting Catalan Numbers

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Abstract

In 1751, Euler proposed to Goldbach the problem of finding in how many different ways a convex polygon of n sides can be divided into triangles. Later, in 1838, Catalan gave a solution to the problem of finding in how many different ways n letters can be "binarily bracketed". For example for the four letters a, b, c, and d there are five possibilities: ((ab)c)d, (a(bc))d, (ab)(cd), a((bc)d) and a(b(cd)). It turns out that the general solutions to BOTH problems are the same: the n-th Catalan Number.

Today, according to Richard Stanley, more than 120 different combinatorial problems have the Catalan numbers as their solution. Many solutions to these problems are done using clever combinatorial bijections, some of which I'll try to illustrate. Moreover, I'll attempt to discuss some interesting properties of these numbers (like closed formulas, recurrence relation and generating function).

Special presentation – 4:50-5:00pm Siobhan Ehle Teach For America