

**Undergraduate Math Club
Winter 2006
2nd floor Nesbitt Common Room
Feb. 2, 4:10-5:00pm
(free pizza and pop, as always)**

Generalized birthday problem

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Abstract

In the classical birthday paradox, the goal is to find the minimum number of people in a room such that the probability of at least two people having the same birthday is at least $\frac{1}{2}$. (The answer is 23.) A natural generalization is this: if we consider a hypothetical year with n days (maybe not 365) and compute the minimum number of people in a room such that the probability of at least two having the same birthday is at least $\frac{1}{2}$, how does this behave as n grows?

This appears to be a very hard problem, so we consider a variant in which we seek to give a simple formula for a number of people (in terms of n) which suffices to ensure the probability of two having the same birthday approaches $\frac{1}{2}$ as n grows. The solution rests on an interesting application of Taylor series to compute a limit. We also treat the case when $\frac{1}{2}$ is replaced with any number between 0 and 1.

