Undergraduate Math Club Winter 2006 2nd floor Nesbitt Common Room February 23, 4:10-5:00pm (free pizza and pop, as always)

Prof. Brian Conrad Rational right triangles

Abstract

The last remaining unsolved problem from Greek mathematics is the problem of determining "congruent numbers": a positive integer is called a "congruent number" if it arises as the area of a rational right triangle (i.e., a right triangle all three of whose side lengths are rational numbers). For example, 6 is a congruent number since it is the area of the 3-4-5 right triangle (and so any square multiple of 6 is a congruent number, by scaling the triangle). Also, 5 is a congruent number because it is the area of the right triangle with side lengths 3/2, 20/3, and 41/6. It is not easy to recognize whether or not a number is a congruent number! For example, 2 is not a congruent number because of Fermat's Last Theorem for exponent 4. In this talk, we explain how to rephrase the congruent number problem in terms of elliptic curves, and show the striking insights that arise in this way.