Undergraduate Math Club
Winter 2008
2nd floor Nesbitt Common Room
Thursday, March 13, 4:10-5:00pm
(free pizza and pop, as always)

Birkhoff's Ergodic Theorem

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Abstract

Consider a gas inside of a closed container. Each molecule of the gas has a velocity, and one can consider both the average velocity of a single molecule in time as it flies around (the ``time average'') and the average velocity taken over all particles in the container at a given time (the ``space average''). Birkhoff's ergodic theorem says that under mild realistic assumptions about the system, the time average for the vast majority of the molecules agrees with the space average of the whole system. I shall state and prove this theorem as a general statement about dynamical systems, informally developing the ideas from measure theory that I need along the way. In the remaining time I will provide some applications of the theorem to number theory and discuss how it can be used to fight crime!