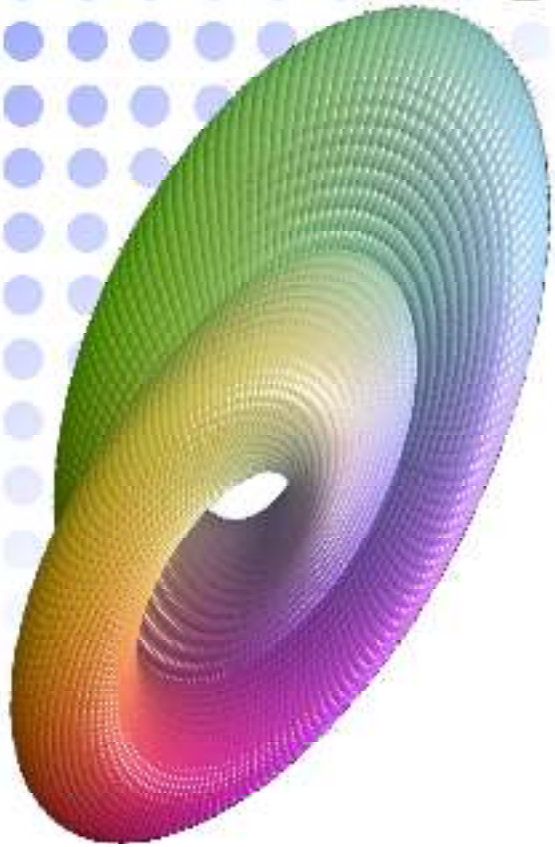
Michigan Math Club

Thursday at 4pm in the Commons

Free Pizza and Pop



How to compute 45 million digits of π

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Abstract for 16 April

By definition, π is the ratio of the circumference of a circle to its diameter. However, if you try to compute π as a decimal expansion from this definition, you won't get very far: even if you had a circle with circumference the size of the entire universe and could measure to within the accuracy of the diameter of a proton, you'd only be able to compute about 49 digits of π correctly. You might try and use your calculus skills and clever formulae with trig functions to do better (as Newton and Leibnitz did) but even if you toiled 15 years in the labor (as Shanks did) you'd be unable to compute more than 707 digits, and you'd probably make a mistake (as Shanks did, at decimal place 527).

In this talk, I'll describe (and prove the validity of) a simple iterative algorithm for computing π efficiently and quickly: after a mere 24 iterations (and but a few hundred arithmetic operations), the algorithm produces over 45 million correct digits of π. This algorithm is based on a powerful connection between elliptic integrals and the arithmetic-geometric mean. I'll only use (clever) calculus in this talk; in particular, no prior knowledge of elliptic integrals will be assumed.