Michigan Math Club

Thursday at 4pm in the Commons
Free Pizza and Pop

What's so great about generating functions?

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Everyone who has taken Math 116 knows about infinite sequences of numbers: a_0, a_1, a_2, …. To study such sequences, it is very common to turn the sequence into a power series $F(x) = a_0 + a_1 x + a_2 x^2 + …$, which is called a generating function. If you've never seen this before, you're probably wondering what is gained by taking a simple list of numbers and turning it into a seemingly complicated function. It turns out that it is often possible to first understand the function $F(x)$, and then use the function to understand the coefficients a_0, a_1, a_2, ….

To illustrate how this works, I will derive a closed form for the Catalan numbers, a famous sequence that appears throughout combinatorics. As another fun example, I will explain how it is sometimes easier to calculate infinitely many integrals at once than one integral at a time!