Michigan Math Club

Thursday at 4pm in East Hall 1360

Free Pizza and Pop

Introduction to Sporadic Simple Groups

Prof. Robert Griess • 24 February 2022

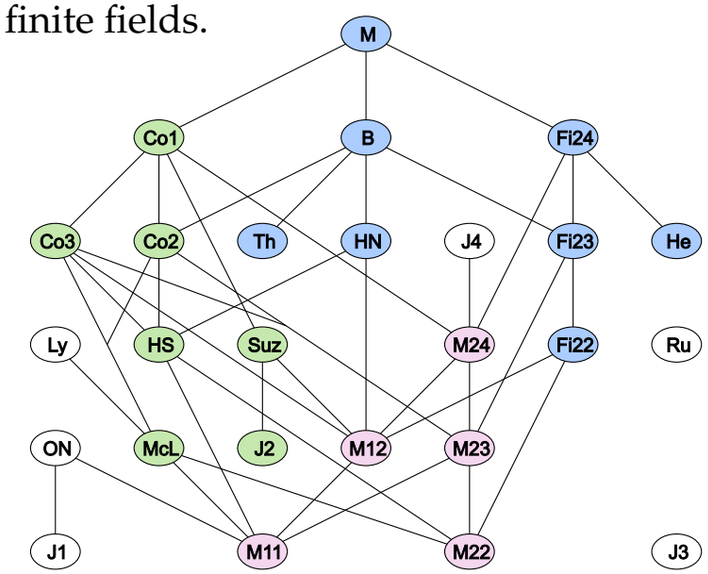
A finite group is made up of simple groups. Think of how a natural number has an essentially unique factorization into primes, e.g.

$$12 = 2 * 2 * 3 = 2 * 3 * 3 = 3 * 2 * 3$$

Many questions about finite groups can reduce to studying properties of the finite simple groups.

Most finite simple groups are relatively easy to describe. The alternating groups of degree n (even permutations on $\{1, 2, \ldots, n\}$) are simple for n at least 5. Certain matrix groups associated to bilinear forms give simple groups, and these are finite groups if matrix entries come from finite fields.

There are just 26 sporadic simple groups. These groups do not belong to infinite families as above. They arise from situations which involve exceptional behavior in number theory, combinatorics and group theory. I shall discuss a few examples. The largest sporadic is the Monster, which involves 20 of the 26 sporadics.



Mathematicians in our department played a role in the story of sporadic groups: Richard Brauer, Donald Higman, Jack McLaughlin and myself. My article on the history of sporadic groups is available at `bit.ly/Sporadic_Life`

This talk will assume background in basic group theory.