# Michigan Math Club

# The sequence $n\alpha$ mod 1

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Take an irrational number and consider the fractional part of all of its multiples. It turns out that the more you stare at this sequence, the more interesting it gets! For example, it is a classical result that this sequence is dense on the set $[0, 1]$. This implies that a line drawn on a torus is dense if it has an irrational slope. A less known fact is that if you were to write out the first $N$ terms in this sequence on $[0, 1]$ and measure distances between consecutive points, you would find at most three distinct distances! When you try to calculate these distances, you find yourself looking at some cursed fractions! In this talk we'll explore some of these questions and the objects that arise while trying to answer them!