Research Profile of Anthony Bloch

Further information may be found on my homepage.

My research includes work on the geometry and dynamics of finite- and infinite-dimensional integrable Hamiltonian systems, work on the geometry and dynamics of nonholonomic systems, and work on the dynamics and control of nonlinear systems, particularly those arising in mechanics. I am currently interested in futher applications to quantum mechanics and biology including connections between neuroscience and locomotion.

I received my Ph.D from Harvard University in 1985 and have received a Presidential Young Investigator Award and a Guggenheim Fellowship among other awards. I have held various visiting postions and was a member of the Institute for Advanced Study. I serve on the editorial boards of a number of journals in dynamics and control.

Some of my recent work on integrable systems has centered on the Toda lattice, a set of interacting particles on the line, but which is linked to a number of areas of mathematical physics. We have studied the links between this integrable flow and gradient flows. We have also shown that the Toda flow is related to the geometry of convex polytopes through the momentum map. This geometrical picture gives us information on the qualitative behavior of the Toda flow. In addition the Toda dynamics can be used to prove geometric results. Recently we have studied the dynamics of the full Toda lattice – a flow on the space of full symmetric matrices rather than the tridiagonal matrices arising in the usual Toda flow. We have also extended a number of these ideas to an infinite-dimensional setting based on the geometry of the group of area preserving diffeomorphisms of the annulus. In this setting the equations become a set of hyperbolic partial differential equations.

This work on smooth flows has applications to numerical problems, such as linear programming and the sorting of numbers.

I am also studying integrability of systems of rigid body type and their connection with optimal control and coupled double bracket equations. These flows are related to the discrete Moser-Veselov flows.

I am interested in various aspects of geometric mechanics. Some of my recent work has focussed on the role of dissipation and forcing in dynamical systems with symmetry and the relationship of these properties to stability and instability. I have also been studying the geometry of nonholonomic systems, that is systems with nonintegrable velocity constraints. Examples of such systems are a disk or sphere rolling on a smooth surface. These systems are not Hamiltonian and have a very rich dynamic behavior. The dynamics can be shown to be governed by the curvature of a connection associated with the constraints and by the system symmetries. I have also been studying the stability of these systems. Nonholonomic systems have remarkable stability properties. Despite the absence of dissipation they may exhibit asymptotic stability. An example that exhibits such behavior is the rattleback top.

In the area of control I am interested in problems of stabilization, motion planning and optimal control. One can show, for example, that nonholonomic systems cannot be smoothly stabilized by static controllers, but that one can develop piecewise smooth controllers for such systems. Also interesting are systems of coupled rigid or flexible bodies. Here one can use internal angular momentum preserving controllers to achieve a desired orientation. These ideas are linked to the notions of holonomy or geometric phase in an appropriate fiber bundle. I have also been studying the integrability of optimal control problems. This approach has shed light on classical geodesic flow problem such as the rigid body flow. Quite a lot of my recent control work has been on computing Lagragian and Hamiltonian control systems by the method of matching which I have developed with my collaborators (see below).

Recently, I have been studying problems in the dynamics and control of quantum systems and have also become interested in neural systems and their relationship to mechanics.

Some recent papers:

Nonholonomic control systems on Riemannian manifolds, *The SIAM Journal on Control and Optimization* **37** No. 1, (1995), 126-148 (with P. Crouch).

Nonholonomic mechanical systems with symmetry, *Archive for Rational Mechanics and Analysis* **136**, (1996), 21-99 (with P. S. Krishnaprasad, J. E. Marsden and R. Murray).

Optimal control and geodesic flows, *Systems and Control Letters* **28** (1996), 65-72 (with P. Crouch).

The Euler-Poincaré equations and double bracket dissipation, *Communications in Mathematical Physics* **175**, 1-42 (1996)(with P. S. Krishnaprasad, J. E. Marsden, and T. S. Ratiu).

The Toda PDE and the Geometry of the Diffeomorphism Group of the Annulus, *Fields Institute Communications* **7** (1996), 57-92, (with H. Flaschka and T. S. Ratiu).

Stabilization and tracking in the nonholonomic integrator via sliding modes, *Systems and Control Letters* **29** (1996), 91-99 (with S. Drakunov).

Double bracket equations and geodesic flows on symmetric spaces, *Communications in Mathematical Physics* **187** (1997), 357-373 (with R. Brockett and P. Crouch).

The energy momentum method for the stability of nonholonomic systems, *Dynamics and Stability of Systems* **13** No. 2 (1998), 123-166 (with D. Zenkov and J. Marsden).

Hamiltonian and gradient structures in the Toda flows, *The Journal of Geometry and Physics* **27** nos. 3-4, (1998) 230-248 (with M. Gekhtman).

Symmetries and integrability of nonholonomic systems, *SIAM J. on Control and Optimization* **36** N0. 6, (1998) 2020-2039 (with P. Crouch).

Stabilization of nonholonomic systems using isospectral flows, *SIAM Journal of Control and Optimization*, **38** (2000), 855-874 (with S. Drakunov and M. Kinyon).

Dynamics of the n-dimensional Suslov problem, *J. of Geometry and Physics*, **34** (2000), 121-136 (with D. Zenkov).

Controlled Lagrangians and the stabilization of mechanical systems I: the first matching theorem, *The IEEE Transactions on Automatic Control* **45** (2000), 2253-2270(with N. Leonard and J. Marsden).

Controlled Lagrangians and the stabilization of mechanical systems II: potential shaping and tracking, *The IEEE Transactions on Automatic Control* **46** (2001), 1556-1571 (with D.- E. Chang, N. Leonard and J. Marsden).

Dispersionless Toda and Toeplitz operators, to appear in the *Duke Mathematical Journal* (with F. Golse, T. Paul and A. Uribe).