Group Homework # 1

This homework is due by **11:59pm on August** 5^{th}. Scans/pictures are accepted as well as digital files (i.e. .pdf) via email/Canvas. Please make sure your submission is printer friendly. Start early!

1. (a) (4 points) For parts (a) and (b) assume that $\vec{r}(t) = \langle \cos(\pi t) + (\pi t)\sin(\pi t), \sin(\pi t) - (\pi t)\cos(\pi t)\rangle$. Sketch a graph of $\vec{r}(t)$ in the plane and label the points at $t = 1, t = 3, t = 3/\pi$.

   (b) (5 points) Find $a_{\vec{T}}$ and $a_{\vec{N}}$ at times $t = 1, t = 2, t = 3/\pi$.

   (c) (6 points) Find the length of the space curve $\vec{r}(t) = \langle 12t, 8t^{3/2}, 3t^2\rangle$ where t ranges between 0 and 1.

2. Throughout this problem $\vec{v} = \langle 1, -2, 3\rangle$ and $P = (2, -1, 4)$.

   (a) (2 points) If $Q = (-5, 2, -1)$, find the vector projection of $\vec{PQ}$ onto $\vec{v}$.

   (b) (8 points) Let ℓ denote the line through P parallel to $\vec{v}$ and consider the point $Q = (-5, 2, -1)$. Find the point R on ℓ such that $\vec{RQ}$ is perpendicular to $\vec{v}$.

3. A golf ball is hit at time $t = 0$. Its position vector as a function of time is given by:

$$\vec{r}(t) = \langle 3t, 2t, -t^2 + 4t\rangle.$$

   Notice that at $t = 0$ the ball is at the origin of the coordinate system. Let the xy-plane represent the ground. At some time $t_1 > 0$ the ball will return to the xy-plane hitting some point $P = (a, b, 0)$.

   (a) (5 points) Compute the velocity and the speed of the ball at an arbitrary time t.

   (b) (5 points) Find the value of the time $t_1 > 0$ and the corresponding coordinates of the point P where the ball hits the xy-plane again.

   (c) (8 points) Set up a definite integral equal to the length of the arc of the trajectory from the origin to the point P. Write a bound on the length of the arc using the *modulus length* bound.

   (d) (12 points) Find the equation of the vertical plane containing the trajectory. What can we say about the velocity vector found in part (a) and the plane?

4. Below, consider the points $P = (1, 1, 1), Q = (-2, 3, 2)$, and $R = (-1, -1, -1)$.

   (a) (5 points) Find the equation of the plane through the points P, Q, and R.

   (b) (3 points) Sketch the triangle $\triangle PQR$ in space on a well labeled graph.

   (c) (8 points) Consider the point $S = (-3, 3, -1)$. The shape defined by $PQRS$ is two triangles, each lying in a different plane. Find the angle between the two planes, and then find the equation of the line which acts as a "crease" between the two triangles. Finally, determine the area of the surface.