**Instructor: Anthony Della Pella**
**Calculus III Group**

# Group Homework # 2

This homework is due by **11:59pm on August** $13^{\textbf{th}}$. Scans/pictures are accepted as well as digital files (i.e. .pdf) via email/Canvas. Please make sure your submission is printer friendly. Start early!

1. (a) (3 points) Suppose the vector $\vec{i}$ in the $xy$-plane is rotated by an angle $\theta$ measured in radians. Draw a diagram representing both $\vec{i}$ and its image under such a rotation and label the components in terms of $\vec{i}, \vec{j}$ and the trigonometric functions.
   (b) (2 points) Now suppose we rotated the vector $\vec{j}$. Again sketch both the original vector and its image on a well labeled diagram.
   (c) (5 points) Suppose a vector $\vec{v} = v_1\vec{i} + v_2\vec{j}$ is rotated by $\theta$. Use the answers from parts (a) and (b) above to find the rotated vector $\vec{v}(\theta)$.
   (d) (6 points) Consider $\vec{v}$ as above. Find $\vec{w} = \dfrac{d\vec{v}(\theta)}{d\theta}$ at $\theta = 0$ using both the limit definition of the derivative, and derivative rules for vector valued functions. Explain why the dot product $\vec{w} \cdot \vec{v} = 0$.
2. The following parts may be unrelated.
   (a) (3 points) Find the value of $x$ so that the angle between $\langle 2, 1, -1\rangle$ and $\langle 1, x, 0\rangle$ is $\pi/4$.
   (b) (5 points) Suppose wind is blowing an ice road trucker off the icy road. The truck can be thought of as centered at the origin in which case the wind is blowing from the direction of $\frac{\pi}{2} + \frac{\pi}{4}$ radians (which is $135°$ degrees. If the wind is blowing at 25 km/h and the truck is driving at 125 km/h find the true course of the truck assuming that the truck has no friction on the ice. In particular, the trucks resultant direction is the sum of the two relevant vectors. A diagram is helpful here!
   (c) (4 points) Suddenly, the ice breaks 1km in front of the truck. Seeing this, the driver hits his breaks. Find the vector representing the force needed to stop the truck so that the truck doesn't plummet into the water.
3. We will use the following helix: $\mathbf{r}(t) = \langle 4\cos(t), 4\sin(t), 3t\rangle$.
   (a) (4 points) Find the arc length of the helix between $t = 0$ and $t = 4\pi$.
   (b) (5 points) Defining the binormal vector $\mathbf{B} = \mathbf{T} \times \mathbf{N}$, find the binormal vector of the helix. Note that you will need the tangent and normal vectors for the next question.
   (c) (3 points) Sketch the helix and the 3 vectors $\mathbf{T}, \mathbf{N}$, and $\mathbf{B}$ on the graph. What property do these three vectors have?
4. (a) (4 points) Think of a cube in space whose bottom closest left (thought of as the first) corner is at the origin. We can think of using a "3D Printing Pen" and printing the edges of this cube in space. Starting at this first corner, find a vector valued function $\mathbf{r}(t)$ which traces out the rest of the cube.
   (b) (2 points) Thinking back to your vector valued function from the above part, can you do this more efficiently? If so, write a new function that uses strictly less material. If not, provide brief justification why you can't do better than your solution from above. Looking up Hamiltonian Cycles may help you with this part.
   (c) (3 points) Now suppose you drew in the diagonal from the first corner to the polar opposite (top far right) corner. What is the angle between this diagonal and the *diagonal* of one of the adjacent faces?