3/25/2020

Online Lecture:

Posted § 4.3 - Soln's to wksht

In GOOGLE DRIVE

Also has corrections to the worksheet problems

- Derivative Rules
- Linear & Quadratic approx.
- Theorems :
    - differentiability
    - Continuity
    - MVT

      ⋮
- Qualitative Derivatives :
    - critical pts.
    - increasing / decreasing
    - inflection points
    - second derivatives (concavity)

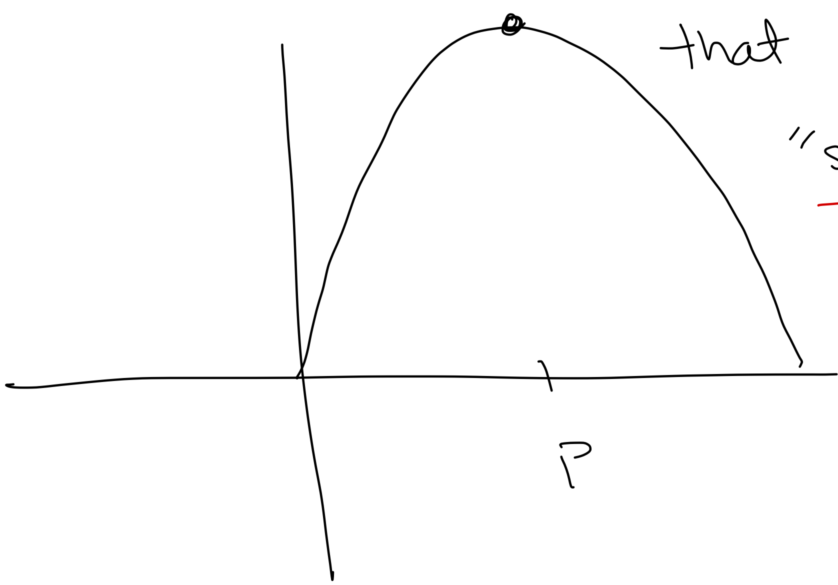
- Optimization :
    - local and global max/min
    - Real world modeling

A not-neccessarly-exhaustive list of exam topics

What is a critical point?

p is a critical point of a function f if

$f'(p) = 0$ or $f'(p)$ is undefined.



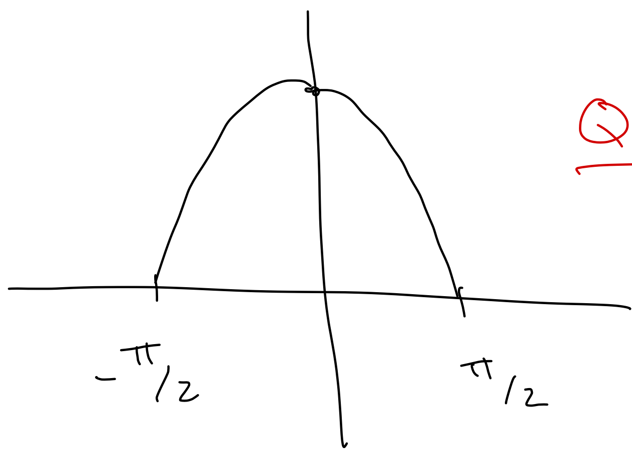
$f'(p) = 0$ means that the function "stops" here.

The more useful interpretation is that critical points are "contenders" for local max and min.

Eg:

Let $f(x) = \cos(x)$, on $(-\pi/2, \pi/2)$

$$f'(x) = -\sin(x)$$



Q: where is f' undefined?

A: the domain of $\sin(x)$ is all real #'s, so $f'(x)$ is never undefined.

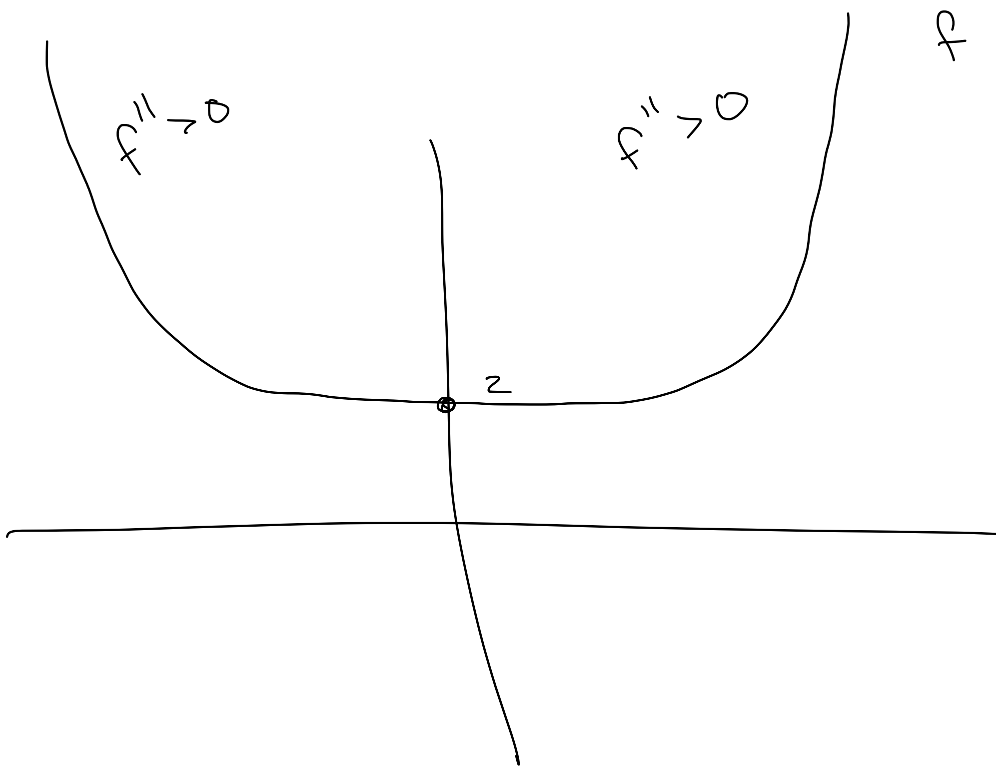
Q: where is f' equal to 0?

A: $-\sin(0) = 0$ so 0 is a critical point.

An inflection point is a point P in the domain of a continuous function where the function changes concavity.

$f = x^8 + 2$

Keep this in mind!

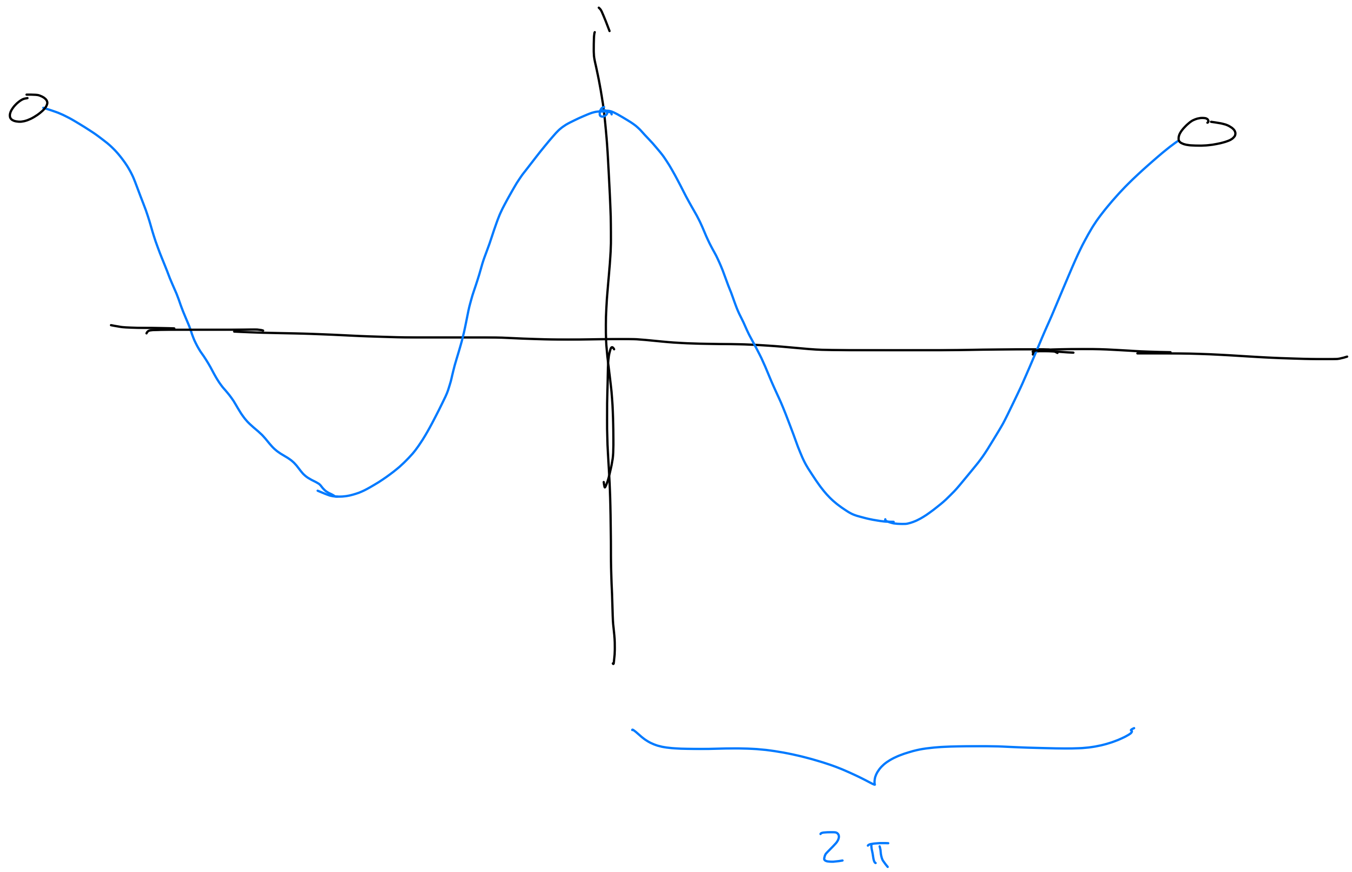


$f''(0) = 0$, but doesn't change concavity here.

- to check whether concavity changes look at second derivatives

Optimization (for functions):

consider $f(x) = \cos(x)$ on $(-2\pi, 2\pi)$



- Find local maxes and local mins
- Find inflection points.

$$f'(x) = -\sin(x)$$

$$f''(x) = -\cos(x)$$

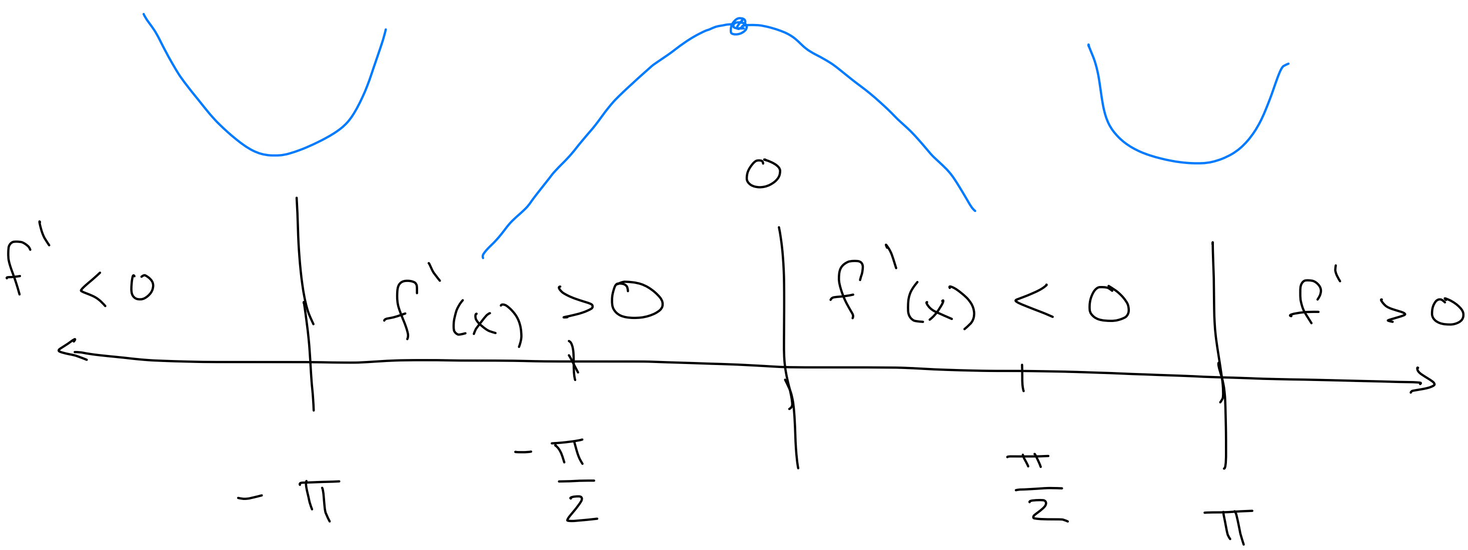
Critical Points:

$$f'(x) = 0$$

$$-\sin(x) = 0$$

$$x = 0, \pm\pi, \cancel{\pm 2\pi}$$

Not in the domain.



$$\left.\begin{aligned} f'(-\pi/2) &= -\sin(-\pi/2) = -(-1) = +1 \\ f'(\pi/2) &= -\sin(\pi/2) = -1 = -1 \end{aligned}\right\}$$ 0 is a local max

Similarly: $-\pi, \pi$ are local mins

We were using an open interval for our domain so

$\lim_{x \to -2\pi} f(x) \overset{?}{>}$ previously found local max.

if $>$ holds then no global max.

local max at 0, $f(0) = 1$

$\lim_{x \to -2\pi} f(x) = 1$ so 0 is a global max.

We found 2 local minima using F.D.T. $\pm\pi$. $f(-\pi) = -1$, $f(\pi) = -1$

$\lim_{x \to \pm 2\pi} f(x) \overset{?}{<}$ previously found local mins

Then no global min.

We can conclude there are 2 global minima at $-\pi$ & π.

We said before

$$f''(x) = -\cos(x)$$

Potential Inflection Points:

$$f''(x) = 0$$

out side of domain

$$-\cos(x) = 0 \quad @ \pm\frac{\pi}{2}, \pm\frac{3\pi}{2} \cancel{, \pm\frac{5\pi}{2}}$$

Go to 115 webpage!

Winter 2020 Exams → Exam 2 →

justification

$-\cos(0) = -1$

$\cos(\pi)$

	$-3\pi/2$		$-\pi/2$		$\pi/2$		$3\pi/2$	
$f''<0$		$f''>0$		$f''<0$		$f''>0$		$f''<0$

<u>Conclusion!</u> $\pm\frac{3\pi}{2}$, $\pm\frac{\pi}{2}$ are indeed inflection pts b/c second derivative (concavity) changes sign.