Exam: Monday April 6th

§ 4.5

can be units of labor!

independent var: q - quantity

$$\begin{cases} \text{Cost} = C(q) \to \text{cost at quantity } q \\ \text{Revenue} = R(q) \to \text{Revenue at quantity } q. \end{cases}$$

$\pi(q) \to$ profit at quantity q.

$\parallel$

Revenue - Cost $= R(q) - C(q)$

utilize Ch. 2!

$R'(q)$ = Marginal Revenue ← Derivative

$C'(q)$ = Marginal Cost ← Derivative

Utilize Ch. 4:

Marginal Revenue & Marginal cost allow us to optimize profits.

Ch. 2:

Interpretations:

"how much R changes when we increase by 1 unit of q"

approximating $R'(q)$, $C'(q)$, $\pi'(q)$ using "real world" ideas.

$$C'(10) \approx C(11) - C(10) \approx \frac{C(10.1) - C(10)}{10.1 - 10}$$

Benefit was that we could get a feel for things in terms of their real world "units".

Chapter 4. Allowed us to max and minimize functions.

- $\pi(q)$ is a function of quantity we would like to optimize our profit!

→ optimization relies on finding critical pts.

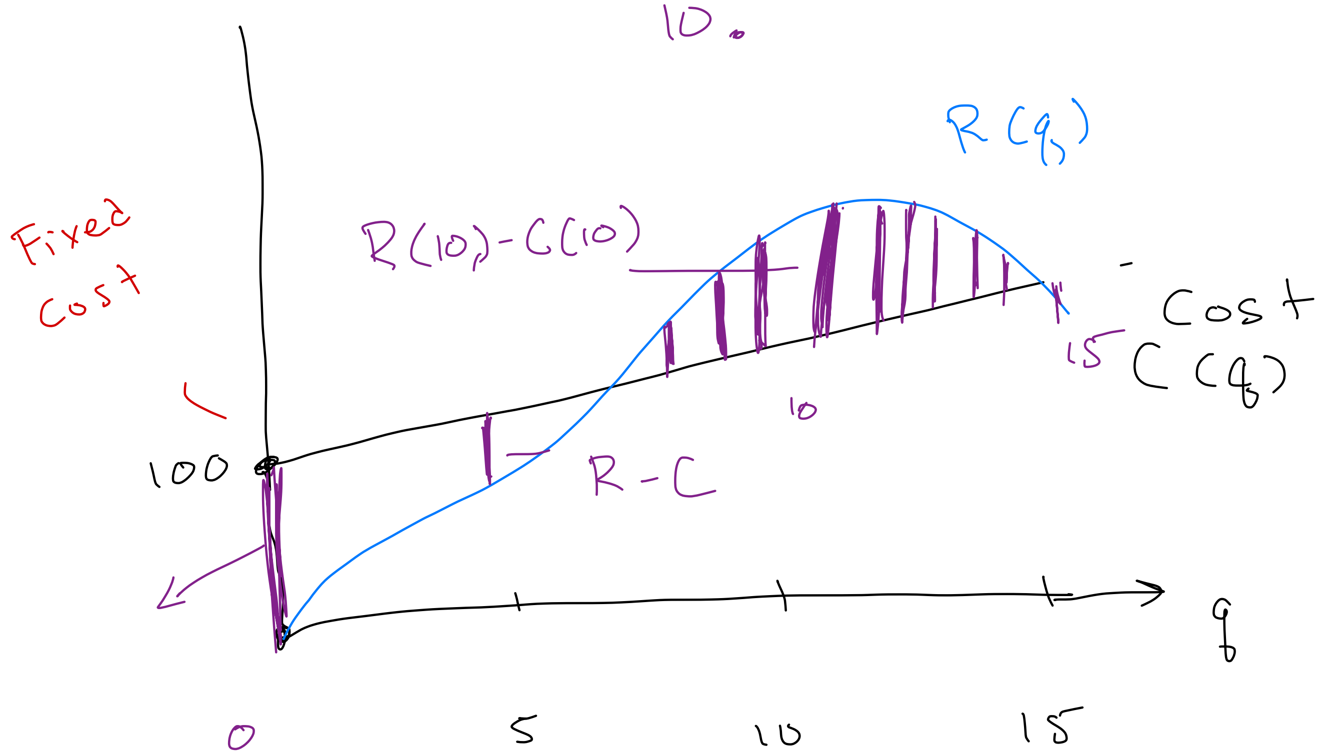
$\pi'(q) = 0$, $\pi'(q)$ undefined

$$\pi'(q) = R'(q) - C'(q) = 0 \rightsquigarrow R'(q) = C'(q)$$

Once we've set marginal revenue = marginal cost then maximization boils down to performing a first derivative test.

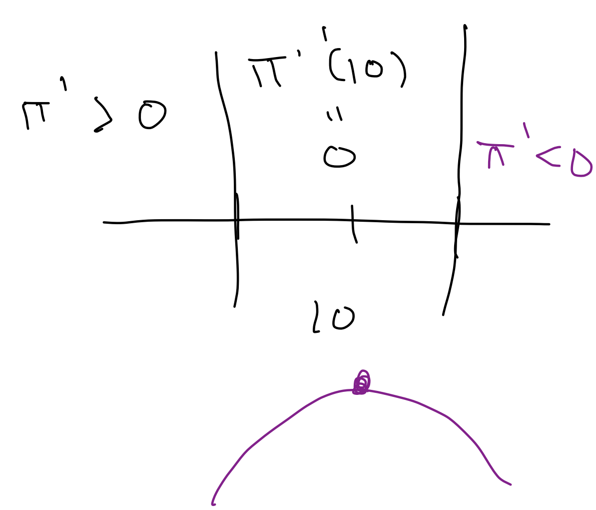
Eg: In practice π could be maximized at any of 0, 10, 15

BUT: in our case only at 10.



Derivative Geometrically:

slope of tangent line.



2 subtle points when doing this

- Always check your endpoints
- Perform an HONEST sign test / first derivative test

Eg:

Revenue:

$$R(q) = 20q - \frac{1}{q}$$

as $q \to \infty$

$R(q) \to \infty$

$$C(q) = 10q$$

as $q \to 0$

$R(q) \to -\infty$

we make $20q - \frac{1}{q}$ dollars when q units are produced.

But it costs $10q$ dollars when q units are produced

Fix domain to be $q \in [1, 10]$.

Ensures π' not undefined.

optimize $\pi(q)$:

$$\pi'(q) = R'(q) - C'(q) := 0 \iff R'(q) = C'(q)$$

$$R'(q) = 20 + \frac{1}{q^2}$$

$$C'(q) = 10$$

Finding Critical Points

Set

$$R'(q) = C'(q)$$

$$20 + \frac{1}{q^2} = 10$$

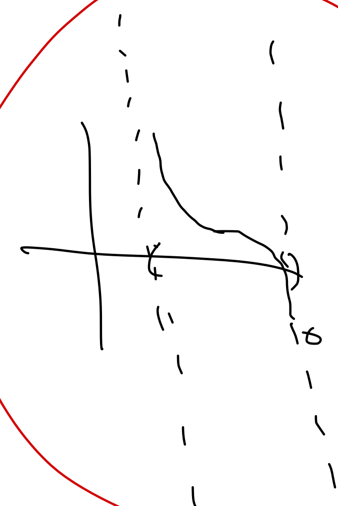
$$-10 = \frac{1}{q^2}$$

$$q = \pm\sqrt{-1/10}$$

Not a real #

Not in [1, 10]

Have no extrema

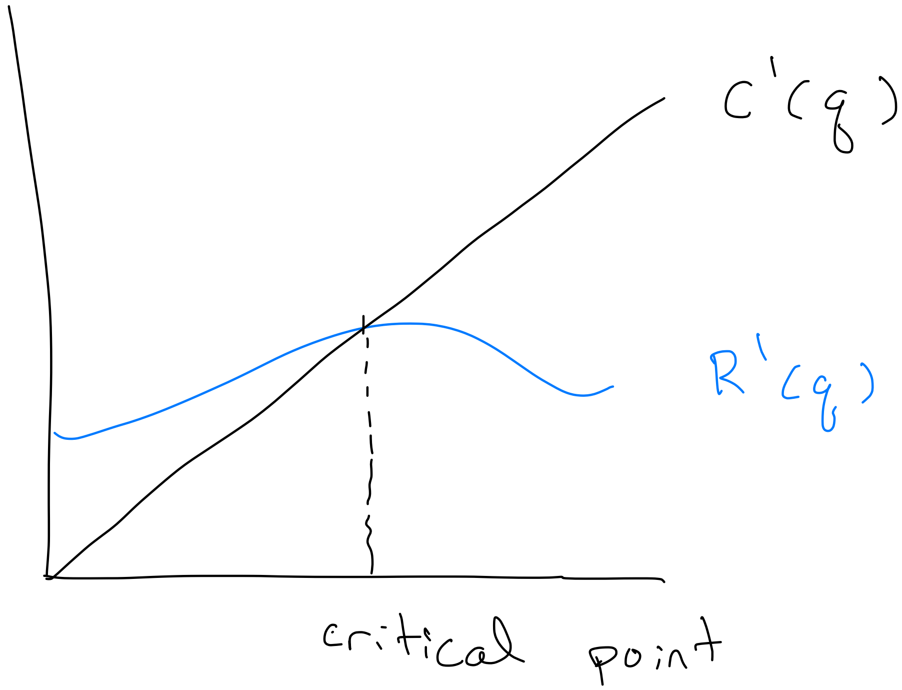


$$\pi(1) = 20 - \frac{1}{1} - 10 = 9$$

$$\pi(10) = 200 - \frac{1}{10} - 100 = 99 + \frac{9}{10}$$

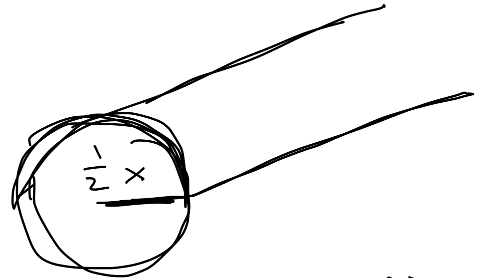
Global max

variant on previous exercise.



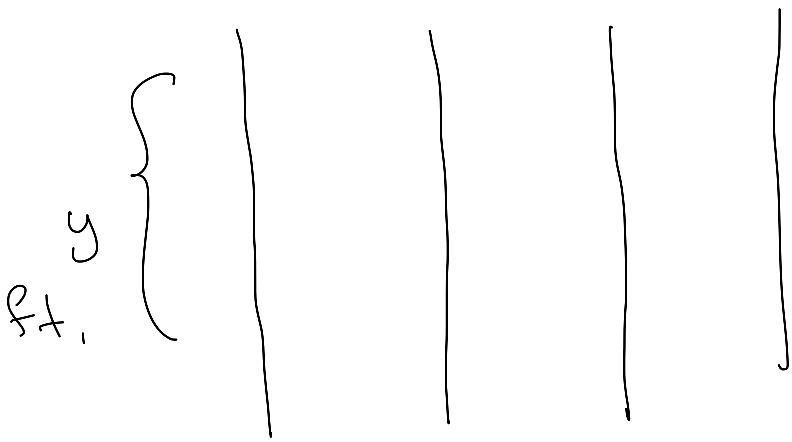
Review:

Q3 - Ex 2 - W18



$\frac{1}{2}\left(2\pi\left(\frac{1}{2}x\right)\right)$

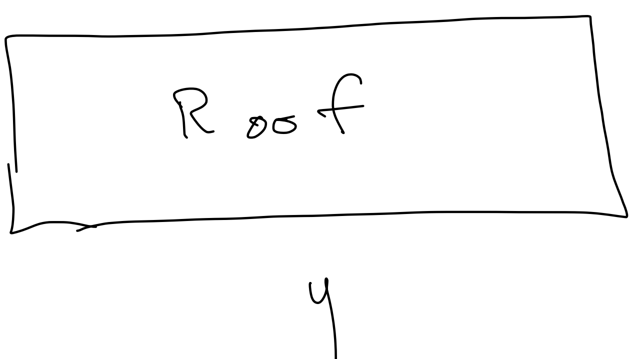
Total is $\$5{,}000$.



4 legs

$\$25$ / ft.

$4 \cdot y \cdot 25 \rightarrow$ amt spent on legs



$\frac{1}{2}$ (circumference)

$\frac{1}{2}(\pi x)$

SA of roof = $\frac{1}{2}\pi x y$

$$\$5{,}000 = \underbrace{\$25 \cdot y \cdot 4}_{\text{legs}} + \underbrace{\left(\frac{\pi}{2}x\right)\$40}_{\text{roof}}$$

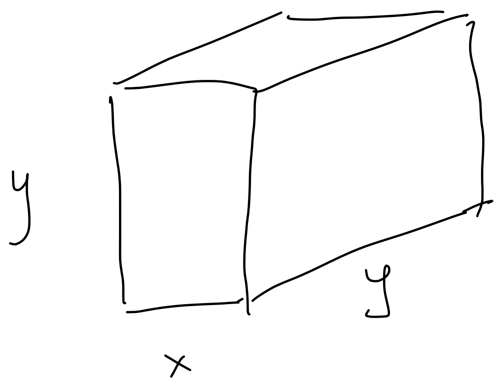
a)

$$\frac{5000 - \left(\frac{\pi}{2}\right)40x}{100} = \cancel{25}\, y \cdot \cancel{4}$$

$$y = \frac{5000 - 20\pi x}{100}$$

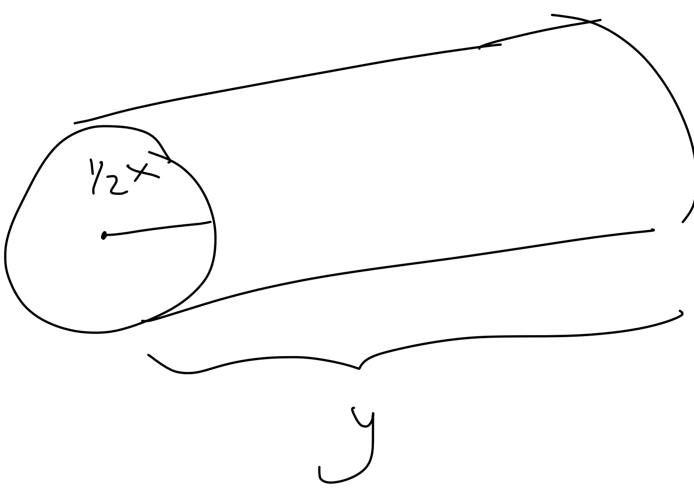
b) Formula for the volume enclosed:

rectangular box:



Volume = $x y^2$

cylinder (half of this)



volume $= \frac{1}{2}\left(\pi\left(\frac{1}{2}x\right)^2 y\right)$

$$V(x) = x\left(\frac{5000 - 20\pi x}{100}\right)^2 \qquad \frac{\pi}{8}x^2\,\frac{5000 - 20\pi x}{100}$$

c) "sides" $= x \geq 5$

"height" $= y \geq 8$

<u>Domain:</u> $V(x) \rightsquigarrow$ function of x

$5 \leq x \leq \frac{210}{\pi} \approx 66.87$

$$8 \leq y \iff \underbrace{8 \leq \frac{5000 - 20\pi x}{100}}$$

$$800 \leq 5000 - 20\pi x$$

$$x \leq \frac{4200}{20\pi}$$

2.5 - using 2^{nd} derivatives:

concavity

2.6 - Differentiability:

- functions need to have continuous derivative

exam problems!

$$f(x) = \begin{cases} Ax^2 & 1 \le x < 3 \\ e^{Bx} & 3 \le x \le 5 \end{cases}$$

at $x=3$ what happens.

Ch. 3:

Gateway

- LINEARITY $\rightarrow$ $\frac{d}{dx}(af(x)+g(x)) = a\frac{d}{dx}f + \frac{d}{dx}g$
- powers
- product \ quotient
- Chain rule
- Exponential & log $2^x \mapsto \ln(2)\, 2^x$
- Trigonometric
- Inverse function $\rightarrow$ on card
- Implicit derivatives

3.9 - Linear (and Quadratic): Approximation

$f(x) \approx f(a) + f'(a)(x-a) + \frac{f''(a)}{2}(x-a)^2$

near a

on card if you don't know it!

3.10 - Theorems:

- MVT
- EVT

Be careful of hypothesis!

Exam! multiple choice

MVT: $[a,b]$ continuous

(a,b) differentiable

4.1 - 1^{st} & 2^{nd} derivs.

critical & inflection points

$f'=0$, f' DNE

concavity changes

4.2 & 4.3 - Optimization & Modeling

1^{st} derivative test

4.5 - Applications to (marginality. / Economics)

<u>Coaching</u> <u>Note</u>:

- Tonight: practice

- I'll collect them until

  Sat 7:00 pm, Grade Sun 7:00 PM.