Team Homework 1

SOME GUIDELINES FOR YOUR FIRST ASSIGNMENT:

- You should read and attempt problems before meeting with your team to discuss them. Even if you aren't able to obtain all the answers, being prepared during the team meetings helps your group work more efficiently during the meeting.
- Don't be discouraged if you cannot solve most of the problems on your own — this is perfectly normal. This is part of why you are being assigned to work on these assignments as a group; make sure to discuss your questions and ideas with your teammates.
- If your team is having trouble with a particular problem, try visiting either study table or office hours with your teammates to get help.
- Make sure *everyone* is involved and no-one feels excluded during the meetings. If you notice someone is shy, actively encourage them to contribute to the group!
- Ask your teammates to explain the reasoning behind their answers if you don't understand it. Remember that all members of the team are responsible for this assignment, and everyone should be on board with what the team turns in.
- Write up your final solutions neatly, and make sure your explanations are clear and complete.
- Make sure you go over the **Team Homework Tutorial** on the course website:

`https://instruct.math.lsa.umich.edu/support/teamhomework/`

1. Let M and F be the number of male and female members (respectively) of a gym t months after January 2015. Some values of M and F are recorded in the table below.

t	1	3	4	6	9	11
F	321	265	210	225	265	208
M	225	210	320	265	208	185

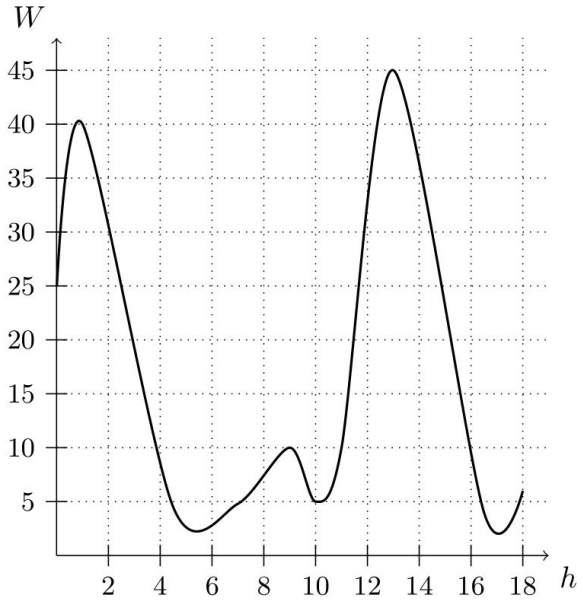
(a) There are multiple possible right answers for this question – you're encouraged to *respectfully* engage your group mates! Be sure to carefully justify your answer.

(i) Is M a function of F?

(ii) Is F a function of M?

(iii) Is F a function of t?

(b) The manager of the gym closely monitors the water consumption W (measured in thousands of cubic feet) in the gym each day. Let $G(h)$ be the amount of water consumed (measured in thousands of cubic feet) in the gym h hours after it opens at 5 a.m. The graph of $W = G(h)$ is shown below.



(i) For what values of h is the function $G(h)$ increasing? You will need to make some estimations for this part.

(ii) Find the average rate of change of $G(h)$ for $1 \leq h \leq 10$. Be sure to include units.

(iii) Give a practical interpretation of your answer in the previous part.

2. Carlos and Mallory are planning to go hiking for a few days, and have already packed all the necessary supplies except for their food and water. Each day during the hike, they'll need 1 kg of food and 5 kg of water. Let F and W be the total mass (in kg) of the food and water respectively that they decide to bring with them.

   (a) Assume that the food they bring along will last for the same number of days as the water they bring along. Write an equation of the form $W = g(F)$ that expresses W as a linear function of F.

   (b) Mallory and Carlos together have a total of 30 L (litres) of backpack space. They'll need 1 L of backpack space for each 1 kg of water and 0.9 L of backpack space for each 1 kg of food they carry. Using this information, write another equation of the form $W = h(F)$, expressing W as a linear function of F.

   (c) What is the slope of $h(F)$, and what does it mean in practical terms?

   (d) Show that the graphs of $W = g(F)$ and $W = h(F)$ intersect at some point (F,W). Determine this point and then give a practical interpretation of what this means.

   (e) How many *full* days can they hike for?

3. Alice, Bob and Eve are each organizing their own office party, which will be catered by Leonardo's restaurant.

   (a) Alice, Bob and Eve had trouble finding the restaurant's rates online, so they each had to call the restaurant to get a quote.

   - Alice needs to feed 100 people at her party, and was told that this will cost $1,800.
   - Bob needs to feed 75 people at his party, and was told that this will cost $1,300.
   - Eve needs to feed 90 people at her party, and was told that this will cost $1,500.

   Let $C(p)$ be the amount (in dollars) that the restaurant charges to cater a party with p people. Find the average rate of change of the function $C(p)$ on the interval [75,90] and on the interval [90,100].

   (b) Based on your answer in part (a), explain why $C(p)$ is not linear.

   (c) Alice, Bob and Eve later meet with Malik, a mutual friend familiar with the restaurant. They told him about the quotes they got from Leonardo's for the office parties. Malik, a former Math 105 student, explains that there must have been a mistake in the price that one of them was quoted, since *the amount Leonardo's charges is a linear function of the number of guests at each party*.

      (i) Let $A(p)$ be the amount Leonardo's should charge to cater a party with p people, if we assume that Alice's quote was incorrect and the other two quotes were correct. Find a formula for $A(p)$ in terms of p using only the quotes given to Bob and Eve.

(ii) Let $B(p)$ be the amount Leonardo's should charge to cater a party with p people, assuming Bob's quote was incorrect while the other two quotes were correct. Find a formula for $B(p)$ in terms of p using only the quotes given to Alice and Eve.

(iii) Let $E(p)$ be the amount Leonardo's should charge to cater a party with p people, assuming Eve's quote was incorrect while the other two quotes were correct. Find a formula for $E(p)$ in terms of p using only the quotes given to Alice and Bob.

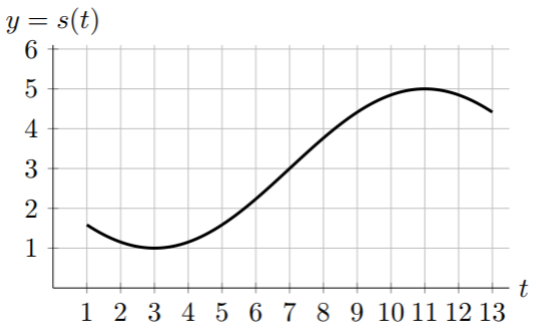
(d) Interpret the vertical intercepts of $A(p)$, $B(p)$ and $E(p)$ in practical terms. Use this to explain whose quote was incorrect.

(e) Is the incorrect quote higher or lower than what the person should actually be paying?

4. An apple farmer wants to assess the damage done by squirrels on the trees in his orchard. in order to do so he installs cameras on a couple of small drones to film the damage done by the vermin. Let $f(t)$ and $s(t)$ be the height above the ground (in feet) of the first and second robot t seconds after they started recording.

(a) Let $f(t) = 4 - 3\cos\left(\frac{\pi}{5}t - \frac{2\pi}{5}\right)$. Find the time(s) at which the first robot is 6 feet above the ground for $0 \leq t \leq 12$. Your answer(s) should be *exact*. Show all your work.

(b) The graph of the sinusoidal function $s(t)$ is shown below only for $1 \leq t \leq 13$. Find a formula for $s(t)$.



(c) Assuming the formula from part (b) suppose now that we want to shoot time-lapse footage. In particular, we wish to view our video at $16\times$ the normal speed. Find a formula representing the height above the ground for our second drone when viewing the faster video (call this quantity $S(t)$).

(d) Again, suppose we speed up the video, this time by a factor of 4. Graph the height of the first drone in this video (denoted by $F(t)$) on a well labeled diagram.

5. A new video is released and a few hours later it goes viral. The number of views, in thousands, of the video t hours after it goes viral is given by the function $v(t)$. For the first 24 hours, the number of views of the video is increasing exponentially, reaching 50,000 views 12 hours after going viral and 120,000 views 24 hours after going viral. After that, (during the second 24 hours), the video is gaining 10,000 views every 3 hours.

   (a) Find a piecewise defined formula for $v(t)$ for $0 \leq t \leq 48$. Show all your work.

   (b) Find the hourly percentage growth rate of $v(t)$ during the first 24 hours. your answer should be given as a percentage accurate up to the first two decimals.

   (c) During the third day, the number of views of the video is not given by a nice formula, but it is known that the difference quotient of views between hours 52 and 53 is given by $\dfrac{v(53)-v(52)}{53-52} = 2$. What are the units of the difference quotient (2)?

   (d) Plot the number of views on a well labeled *log-scale graph* for the first 72 hours. That is, the vertical axis (corresponding to the dependent variable) should be a logarithm of the number of views. Note that the part of the graph for the third day is open to interpretation, but the properties implied by part (c) should be present in your graph.