Team Homework 2 – First Draft

Please check Canvas occasionally as there may be a problem or two added to this assignment.

1. Dorraine is a fire poi artist. Poi are weights which are attached to a rope and then swung in complicated patterns; fire poi are soaked in fuel and burn as the artist performs. In her latest trick, Dorraine swings an unlit poi in a vertical circle (i.e., perpendicular to the ground). She places a fire so that the poi catches fire on its way up when it swings $2\pi/5$ radians past the bottom of the circle. The poi is attached to a 97 cm rope. Dorraine holds the other end of the rope at a constant height of 114 cm above the ground as she swings the poi. The poi rotates at a constant rate, making 7 revolutions in 3 seconds. Let $h(t)$ be the height in meters of the poi above the ground t seconds after it catches fire.
   (a) At what height above the ground does the poi catch fire? Your answer should include units.
   (b) Use your intuition for the behavior of the poi to sketch a graph of $h(t)$.
   (c) Write a formula for $h(t)$.
   (d) Compute the value of t for the first four times that the lit poi is 1.83 m above the ground. Give your answers both in exact form and to the nearest hundredth of a second. *Show your work.*
   (e) Use the limit definition of instantaneous velocity to write an expression for the instantaneous vertical velocity of the poi 3.7 seconds after it caught fire. Do not evaluate this limit.

2. Eshe and Fayang perform similar experiments. They each begin with a 300 mL solution of 50 grams of salt dissolved in water.
   (a) Eshe has a large container. She adds distilled (pure) water to her container at a constant rate of 10 mL/sec. Write an expression for the rate of change in concentration in of salt in the solution (with respect to time) at $t = 20$ seconds after she begins her experiment.
   (b) Fayang performs his experiment with a 300 mL container so his salt solution overflows as soon as he begins to add distilled water to his solution. Despite this mess, he too adds distilled water at a constant rate of 10 mL/sec. The amount of salt $q(s)$ in his container is an exponential function of s, where s is the number of seconds since he began his experiment.
      (i) The amount of salt in Fayang's salt solution is 5.684 grams one minute after his experiment begins. Compute the *continuous decay rate*, half-life, and common ratio of $q(s)$.
      (ii) Write a formula for the concentration $b(s)$ in grams/mL of salt in Fayang's solution s seconds after he begins his experiment.
   (c) Assume that (I) Fayang and Eshe begin their experiments at the same time and that (II) Eshe's container is large enough so that it will not overflow.
      (i) Write a formula for the ratio of the concentration of Fayang's salt solution to that of Eshe's u seconds after their experiments begin.
      (ii) What is the long-run behavior of this ratio?

(iii) Use limit notation to write an equation expressing your answer to part ii.

(iv) What is the purpose of each of assumptions (I) and (II)?

3. Angie Neriman and Simon Yusbeck decide decide to enjoy the fall weather by racing each other from the brass block "M" in the center of the Diag along a 2.5 kilometer (2500 meter) route to the Huron River inside the Arb. Let $A(t)$ (respectively $S(t)$) be Angie's (respectively Simon's) distance along the route (in meters) t seconds after they start racing. Angie and Simon are both wearing GPS watches that record data about their race. The table of values for the functions A and S below shows some of the resulting data, rounded to the nearest meter. Note that the data is not always recorded at regular intervals.

t	0	30	60	66	72	105	114	120	135	168	180	198	300
$A(t)$	0	55	119	137	156	226	249	265	302	384	415	463	737
$S(t)$	0	57	120	137	156	225	248	264	303	389	422	473	768

(a) Estimate Angie's instantaneous velocity 2.5 minutes into the race. Be sure to include units.

(b) Estimate $S'(105)$.

(c) Estimate $S'(100)$. Note that $t = 100$ does not appear in the table.

(d) In the context of the problem, what are the units of the quantity $(S^{-1})'(150)$?

(e) Who was ahead 3 minutes into the race? You should provide justification for your answer.

(f) Who was running faster exactly 1 minute and 12 seconds into the race?

(g) Simon describes the race later and says that his average velocity during the entire event was 2.8 meters per second while Angie mentiones that after the first 5 minutes, her average velocity for the rest of the event was 3.1 meters per second. Assuming all values are accurate, determine who won the race. If there is not enough information state why, and what you would need to decide a winner.

4. Vikram takes a non-stop train ride from Chennai straight to New Delhi. Let $g(t)$ be the distance (in km) of Vikram's train from Chennai t hours after his ride begins. Assume that the function g is increasing and invertible, and that g and g^{-1} are differentiable. Several values for $g(t)$ are shown in the table below.

t	0	2	5	6.5	10	11	16	28
$g(t)$	0	132	346	448	691	741	1153	2180

(a) Estimate the instantaneous velocity of Vikram's train 6 hours after his ride begins. Show your work and include units.

(b) Suppose $(g^{-1})'(900) = C$ where C is some constant.

(i) Using the data in the table fing the best possible estimate of C.

(ii) In the above, what are the units on 700?

(iii) What are the units of C?

(c) Let $R(t)$ be the total rainfall (in cm) in New Delhi during the first t hours of Vikram's train ride. Express the following statement as a single mathematical equation: " Over the first 900 km of Vikram's train ride, it rained 2.6 cm in New Delhi".