Extra problems for section 5.5

1. The sum of two positive numbers is 48. What is the smallest possible value of the sum of their squares?

2. The manager of a computer shop has to decide how many computers to order from the manufacturer at one time. If she orders a large number, she has to pay extra in storage costs. If she orders only a small number, she will have to reorder more often which will involve additional handling costs. She has found that if she orders lots of x computers, the storage and handling costs for a year will be C dollars, where

$$C = 15x + \frac{24,000}{x} + 6,000.$$

   How many should she order at one time to minimize her costs?

3. Among all rectangles whose perimeter is 100 meters, find the dimensions of the one with the largest area. What if the perimeter were L meters?

4. The local council is planning to construct a new sports ground in the shape of a rectangle with semicircular ends. A running track 400 meters long is to go round the perimeter. What choice of dimensions will make the rectangular area in the center as large as possible? What should the dimensions be if the total area enclosed by the running track is to be as large as possible?

5. A wire 2 meters long is cut into 2 pieces. One piece is bent into a square for a stained glass frame while the other piece is bent into a circle for a TV antenna. To cut down on storage space, where should the wire be cut to minimize the total area of both figures? Where should the wire be cut to maximize the total area?

6. Some airlines have restrictions on the size of items of luggage that passengers are allowed to take with them. One has a rule that the sum of the length, width and height of any piece of luggage must be less than 158 cm. A passenger wants to take a box of the maximum allowable volume. If the length and width are to be equal, what should the dimensions be and what will the volume be? If the length is to be twice the width, what should the dimensions be?

7. A wire 6 meters long is cut into 12 pieces. These pieces are welded together at right angles to form the frame of a box with square base. Where should the cuts be made to maximize the volume of the box? Where should the cuts be made to maximize the total surface area for the box?

8. I want to make a fish tank with a volume of 2 cubic meters and I would like the base of the tank to be a rectangle twice as long as it is wide. The base and sides of the tank are to be of glass. What shape of tank will use the least amount of glass (and so cost least.)

9. A factory makes cylindrical cans of volume 500 cubic centimeters. If they want to use the least amount of metal, what should they make the diameter and height of the can? If the metal for the top and the bottom of the can costs twice as much as the metal for the sides, what should the dimensions of the can be to minimize the cost?

10. Find the coordinates of the point on the curve $y = \sqrt{x}$ which is closest to the point $(1, 0)$.

11. A farmer is planning to plant a small orchard and is gathering information about the amount of fruit he can expect to harvest each year once the trees are mature. He is advised that if he plants up to 60 trees of a particular type on his plot of land the average harvest from each tree will be about 120 kg, but for each additional tree planted the expected yield will go down by an average of 2 kg per tree, as a result of overcrowding. Naturally he wants to plan for the maximum yield of fruit. How many trees should he plant?

12. The cost of running a small van at a speed of v km/hr is $25 + .01v^2$ dollars per hour. How long would it take for a trip of 100 km at a constant speed of v km/hr, and how much would it cost? How fast should the driver travel in order to minimize the cost?