Math 115 Calculus Exam I
24 September 1997

Department of Mathematics
University of Michigan

Name: ____________________ Instructor: ____________________

Signature: ____________________ Section: ____________________

General instructions: Please read the instructions on each individual problem carefully, and indicate answers as directed. Use units wherever appropriate.

This test consists of 11 questions on 13 pages (including this cover sheet and a blank final page), totalling 100 points. When the exam begins, please count all of the pages of the exam, make sure none of them are missing, and write your name on each page.

Problem	Points	Score
1	6	
2	5	
3	5	
4	12	
5	8	
6	8	
7	8	
8	10	
9	10	
10	12	
11	16	
Total	100	

NO PARTIAL CREDIT SECTION. (Problems 1–4.) No explanation necessary; no need to show work.

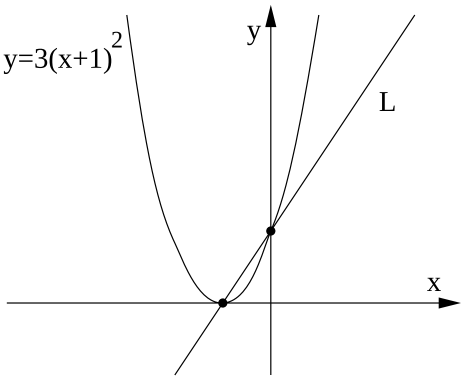
1. (6 points) Suppose $f(t)$ is an exponential function. Complete the following table.

t	1	2	3	5
$f(t)$		8	4	

2. (5 points) Suppose you put \$80000 in a bank account which gains interest at an annual percentage rate (APR) of 10% (or in other words, a nominal rate of 10%), compounded 13 times per year. How much money will you have in your account after 7 years have passed? (Round off your answer to the nearest dollar.)

Answer: $\boxed{}$

3. (5 points) Suppose that the curve below is the graph of $y = 3(x+1)^2$. Complete the equation of the line L shown below. (Note that the graph is not necessarily drawn to scale.)



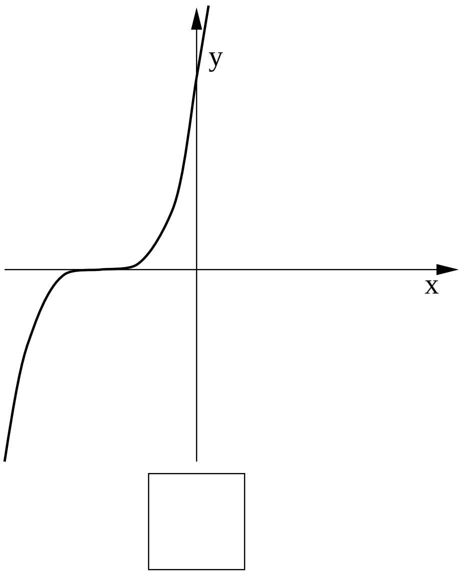
Equation of the line L:

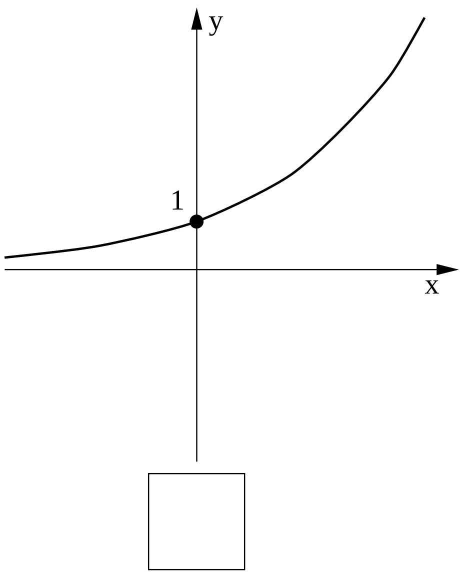
$$y = \boxed{}\,x + \boxed{}.$$

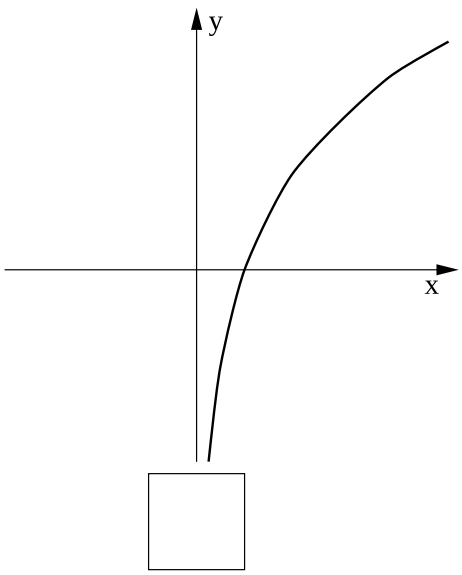
4. (12 points) Consider the following equations:

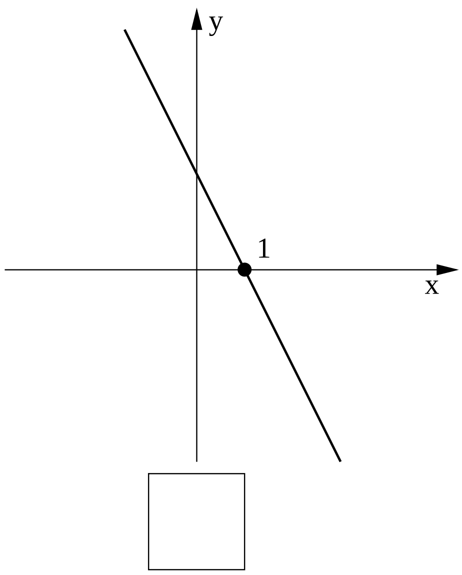
(A)	$y = 729(x+3)^3$	(D)	$y = -\ln x$	(G)	$y = (x-300)^3$	(J)	$y = \log x$
(B)	$y = x^2+1$	(E)	$y = -2x+2$	(H)	$y = 100x-100$	(K)	$y = \cos(2x)$
(C)	$y = 20e^x - 20$	(F)	$y = 2\cdot 3^x$	(I)	$y = \log(x+1)$	(L)	$y = e^{x/1000}$

For each graph shown below, in the box below the graph, write the **single** letter corresponding to the function the graph represents. **Note that the horizontal and vertical scales of each graph may be different.**









SHORT ANSWER SECTION. (Problems 5–10.) Limited partial credit may be possible, and a few sentences of explanation may be required.

In addition, you may be asked to "state your final answer in the form of a complete sentence." This does not mean that you have to explain your work; it just means that your final numerical answer must be in the form of a complete sentence. Example: "Therefore, Georgia and Ira will finish playing at 1:23am."

5. (8 points) On January 1st, 1998 you are offered two jobs. Job A pays 40,000 dollars per year, while Job B pays 50,000 dollars per year. However, starting in 1999, you will recieve a 6 percent raise every year on January 1st in Job A, but only a 3 percent raise every year on January 1st in Job B. What is the first year in which you will earn more on Job A than you would on Job B?

NO EXPLANATION NECESSARY, but show all your work, and state your final answer in the form of a complete sentence.

6. (8 points) The minimum speed required to keep a plane aloft is proportional to the square root of its total weight, including the plane and everything it carries. This minimum speed is called the *stall speed* of the plane.

The C-122A cargo plane weighs 49,000 lbs. when it carries no cargo. When it carries the maximum amount of cargo (28,000 lbs. of cargo, for a total fully loaded weight of 77,000 lbs.), its stall speed is 53 miles per hour. What is the stall speed of the C-122A cargo plane when it is not carrying any cargo? (Round off your answer to the nearest tenth of a mile per hour.)

NO EXPLANATION NECESSARY, but show all your work, and state your final answer in the form of a complete sentence.

7. An all-retro jukebox has six buttons. By each button, there is a sign saying what song each button corresponds to, as shown below:

Button Number	Artist	Song
1	The Cure	"Pictures of You"
2	R.E.M.	"Fall on Me"
3	New Order	"Blue Monday"
4	R.E.M.	"Orange Crush"
5	B-52's	"Rock Lobster"
6	Erasure	"A Little Respect"

(a) (4 points) Is the artist whose song will be played a function of the button pushed? **EXPLAIN** in no more than **TWO** sentences.

(b) (4 points) Is the button pushed a function of the artist selected? **EXPLAIN** in no more than **TWO** sentences.

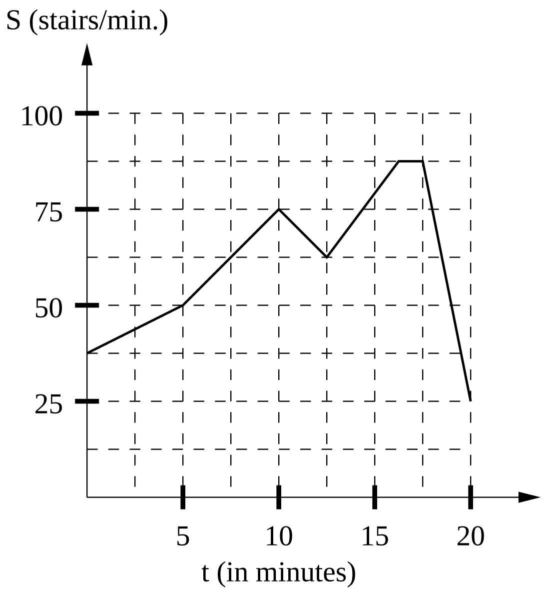
8. (10 points) A colony of bacteria is growing exponentially. At noon on Tuesday, Sept. 16, the colony has 100,000 members, and at 6pm on Tuesday, Sept. 16, the colony has 120,000 members.

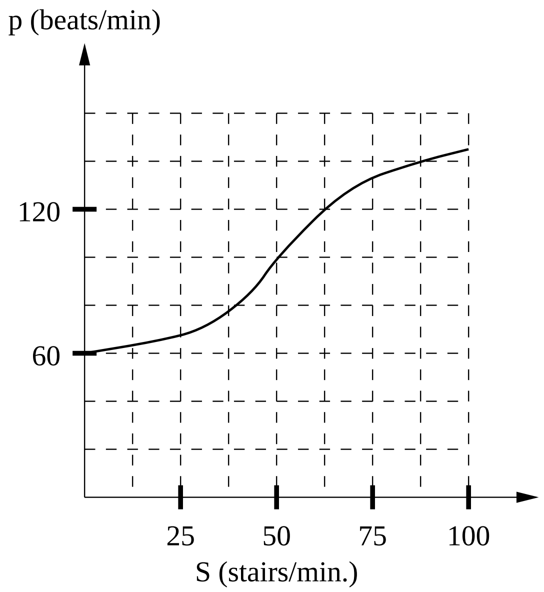
(a) How many members will the colony have at noon on Wednesday, Sept. 17?

(b) When will the size of the colony be three times its size at noon on Tuesday, Sept. 16? (Round off your final answer to the nearest tenth of an hour.)

For both parts, **NO EXPLANATION NECESSARY**, but show all your work, and state your final answer in the form of a complete sentence.

9. A man exercises on a stair-climbing machine for 20 minutes, starting at $t = 0$ minutes. Below, the first graph shows S, the number of (simulated) stairs per minute he climbs, and the second graph shows his pulse rate p as a function of the number of stairs per minute he climbs.



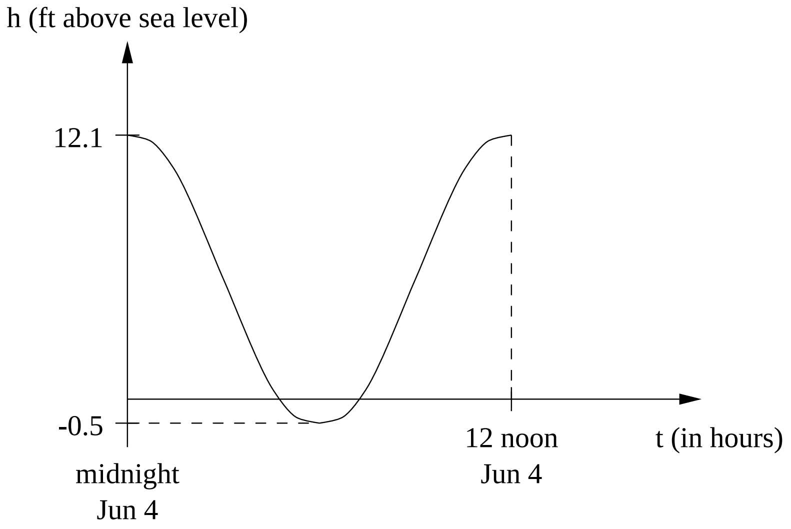


(a) (3 points) Estimate $p(S(5))$. **NO EXPLANATION NECESSARY**, but show all your work.

(b) (3 points) Estimate $p^{-1}(140)$. **NO EXPLANATION NECESSARY**, but show all your work.

(c) (4 points) What does $p(S(5))$ represent in practical terms (i.e., in terms of pulse, steps, etc.)? **EXPLAIN** in no more than **TWO** sentences.

10. Because of the rising and falling tides, the water level at Rockaway Beach on June 4 (between high tides at midnight and noon) varies as sketched in the graph below. Note that the graph is **not** drawn to scale.



(a) (6 points) Let t be the number of hours passed since midnight, June 4. **ASSUMING** that the water level h at Rockaway Beach is given by a sine or cosine curve, find an equation for h as a function of t. **NO EXPLANATION NECESSARY**, but show all your work.

(Problem 10 continued on next page.)

(b) (6 points) If Joey and Dee Dee have carved their initials in the Rockaway Beach pier at the height of 4 feet above sea level, at which time(s) between midnight and 12 noon, June 4, will their initials be above the surface of the water? **NO EXPLANATION NECESSARY**, but show all your work, and state your final answer in the form of a complete sentence. Also, round off your answer(s) to the nearest minute.

ESSAY QUESTION

EXPLAIN your answer using complete sentences. Use graphs (labelled carefully and neatly), tables, and/or formulas in your explanation if possible.

11. On space station Mir, Svetlana is performing an experiment comparing how two different types of fungi (blue and red) grow in space. Over the first 5 days of her experiment, she records the following data about the number of fungi cells (measured in millions) in each fungal colony. (All data is rounded off to 3 decimal places.)

Days passed	Blue cells (millions)	Red cells (millions)
0	11.136	12.037
1	12.563	12.398
2	13.990	12.770
3	15.417	13.153
4	16.844	13.548
5	18.271	13.954

ASSUME that the number of fungi in each colony can be modelled accurately by either a linear function, a quadratic power function, a cubic power function, or an exponential function. (Both functions may be of the same type.)

(a) (10 points) Let t be the number of days passed since the beginning of the experiment. For each colony, find an equation that describes the population growth of that colony, and **JUSTIFY** your choice of equation. (You may continue your answer on the back of this sheet.)

(Problem 11 continued on next page.)

(b) (6 points) Using your equations from part (a), decide which colony will have the bigger population in the long run, and **JUSTIFY** your answer.

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