

## Math 423 Winter 2001 Final Exam

*Each question worth 10 points*

(1) An 8% coupon bond pays coupons on February 20th and August 20th. The bond has a face value of \$100 and matures on August 20th 2002. It is now April 20th 2001 and the bond is being traded for \$102. Estimate the yield on the bond.

(2) Companies X and Y have been offered the following rates per annum on a \$7 million 12-year loan:

	Fixed rate	Floating rate
Company X	7.5 percent	LIBOR+2.0
Company Y	6.3 percent	LIBOR+1.2

Company X requires a fixed-rate loan; company Y requires a floating-rate loan. A bank, acting as intermediary, designs a swap that nets the bank 0.15% per annum and appears equally attractive to X and Y. Find the rate of interest X pays on the fixed rate loan.

(3) A European put option on a non-dividend stock is priced at \$3.24. The current stock price is \$45, the strike price of the option is \$44, the expiration date of the option is in 4 months and the risk-free interest rate is 6% per annum, continuously compounded.

- (a) Find an upper bound for the corresponding American put option.
- (b) Design a hedging strategy showing that your answer to (a) is implied by the no arbitrage condition.

(4) A dividend paying stock has volatility of 30% per annum and its current price is \$28. The risk-free interest rate is 6% per annum, continuously compounded. The stock is due to go ex-dividend in two months with an expected dividend of 40 cents. Find the Black-Scholes price of a European call option on the stock with strike price of \$29 and time to maturity of six months.

(5) Suppose that a portfolio is worth \$88 million and the *S&P* 500 is at 1100. If the value of the portfolio mirrors the value of the index, what options should be purchased to provide protection against the value of the portfolio falling below \$79 million in eight months's time? Each contract is on 100 times the index.

(6) The current exchange rate between the U.S. dollar and sterling is \$1.672. The rate of interest in the U.S. is 5.7% and in England it is 6.2%. An at the money European put option on 1 pound sterling with expiration date in 9 months costs 12 cents. Find the annual implied volatility of the sterling-dollar exchange rate. (Note "at the money" means the strike price is the futures price).

(7) A trader in the U.S. has a short position in a nine month futures contract on 1 million German marks. The current rate of exchange is 1 mark = 53.27 cents. The risk-free interest rate in Germany is 4.25% per annum and in the U.S. it is 6.25% per annum, both with continuous compounding.

(a) Find the delta of the trader's position.

(b) Find the theta of the trader's position.

(8) A company has a portfolio consisting of 10,000 shares of Lucent technology stock and 15,000 shares of IBM stock. Lucent stock is currently priced at \$75 and IBM at \$105. The daily volatility of Lucent stock is 2.3% and of IBM is 1.6%. The coefficient of correlation between their returns is 0.65.

(a) Find the 10 day 95% value at risk of the portfolio.

(b) By how much does diversification reduce the VaR?