Math 523 Final Fall 2000

(1) Suppose the aggregate claims variable S for a policy is compound with frequency variable N and severity variable X. The severity variable X is gamma with parameters (r, λ), $r = 8$, $\lambda = 2$. The frequency N satisfies $E[N] = 10$ and $E[N^2] = 200$.
Find (a) $E[S]$. (b) $var[S]$.

(2) An insurance company has a portfolio of policies on 200 independent identically distributed risks X_i, $i = 1, \cdots, 200$. The risk X_i is exponential with a mean of 3. The policy on X_i is stop-loss insurance with deductible 2. The company wants to be 95% sure that total premiums for the portfolio exceed total claims. Find the premium per policy the company should charge.
(Use the fact that $P(Z \leq 1.645) = 0.95$ for a standard normal variable).

(3) An individual can make at most one claim each year. The expected annual claim frequencies of the entire population of insureds are uniformly distributed over the interval $(0, 1)$. An individual's expected annual claim frequency is constant through time. A particular insured had 1 claim during the prior three years. Using Buhlmann credibility, what is the estimate of this insured's future annual claim frequency?

(4) An individual automobile insured has a claim count distribution per policy period that follows a Poisson distribution with parameter Λ. For the overall population of automobile insureds, the parameter Λ has a probability density function,

$$f(\lambda) = 4\lambda^2 \exp[-2\lambda], \quad \lambda > 0.$$

One insured is selected at random from the population and is observed to have a total of 2 claims during 3 policy periods. Determine the mean and variance of the number of claims that this same insured will have during the fourth policy period.

(5) Jane and Michael go to the post office together. Jane joins the queue to send parcels while Michael joins the queue to buy stamps. Suppose Jane is the next person waiting to be served and Michael has 2 people waiting in front of him. The mean time for a person to spend at the parcel counter is 5 minutes and the mean time at the stamps counter is 2 minutes.
(a) Find the probability that Michael will be served within 3 minutes.
(b) Find the probability that Michael will be served before Jane.

(6) An economist models the employment history of a typical individual by a 2 state continuous time Markov chain. He assumes the average time the person holds a job is 2 years and the average time the person is unemployed before he gets a new job is 1 month. Find, according to this model, the expected number of months a person is unemployed during a 10 year period. Assume the person starts off having a job.

(7) A compound Poisson process has claim variable X which is uniform in the interval $[0, 3]$. Let L be the maximal aggregate loss on the policy, and θ be the security loading.
(a) Determine an optimal value of θ so that $E[L] \leq 5$.
(b) Determine an optimal value of θ so that $P(L > 5) \leq 0.1$.

(8) An insurer is carrying an insurance portfolio which is a compound Poisson process with exponential claim variable. The insurer's security loading is 30% and its current probability of default is 20%. The insurer wishes to reduce the probability of default to 5%. To do this the company purchases proportional reinsurance. Find the proportion of the portfolio that needs to be ceded to the reinsurer, assuming that the reinsurer's security loading is also 30%.