Math 523 Midterm Fall 2000

(1) An insurance company carries a risk X which has pdf $f(x)$, $x \geq 0$,

$$f(x) = \begin{cases} C, & 0 < x < 10, \\ \frac{1}{10}e^{-x/10}, & x > 10. \end{cases}$$

(a) Find the value of C.
(b) Suppose the company offers stop-loss insurance on the risk with deductible 10. Find the pure premium on the policy.

(2) An insurance company models annual medical insurance claims from a large corporation by a compound Poisson variable. Suppose the insurance company assumes that mean annual claim frequency is 2500 and the severity variable is gamma with parameters (r, α), where $r = 4$, $\alpha = 0.01$. The insurance company wants to charge a premium that is large enough so that the probability annual claims exceed the premium is less than 5%. Using the normal approximation, estimate the minimum value of the premium. [Use $P(Z < 1.645) = 0.95$ for a standard normal variable Z.]

(3) The annual frequency variable N for a claims model has moment generating function $\Phi_N(t)$ given by,

$$\Phi_N(t) = \left[\frac{1}{3 - 2e^t} \right]^{50}.$$

(a) Find the mean of N
(b) Find the probability $P(N = 1)$.

(4) An aggregate claims variable is compound Poisson with expected number of claims equal to 4. Individual claim amounts can take only the values 3 and 9, with equal probability. Calculate the expected aggregate claim if an aggregate deductible of 10 is applied.

(5) In a compound Poisson model the number of claims necessary to establish full credibility for frequency is 1000. Suppose the severity variable is exponential with mean 5. Find the credibility of 1000 claims for pure premium.

(6) An insurance company classifies individuals into 2 categories, A and B, with the probability of an individual being type A equal to 2/3. In a single year an individual can make either no claim or a claim of 1. The probability of a type A individual making no claim is 3/5 and the probability that a type B individual makes no claim is 7/8.
(a) Find the a-priori annual pure premium for an individual.
(b) Suppose the individual makes no claim in the first year of the policy. Determine his Bayesian premium for the second year.