Math 523 Final Winter 2000

(1) Suppose the aggregate claims variable S for a policy is compound with frequency variable N and severity variable X. The variable X satisfies $E[X] = 7$, and $\text{var}[X] = 18$. The frequency N is geometric with $E[N] = 20$.
Find (a) $E[S]$. (b) $E[S^2]$.

(2) A company has decided that its standard for full credibility of frequency in a Poisson claims process is 1067 claims. With this standard, using classical credibility theory, the company is 95 percent certain that observed frequency is within r percent of its actual value. Find the value of r. You are given that for a standard normal variable Z, $P(Z < 2.17) = 0.985$, $P(Z < 1.96) = 0.975$, $P(Z < 1.645) = 0.950$, $P(Z < 1.282) = 0.900$.

(3) A life-insurance company covers 100 independent lives for 1-year term life insurance. The benefit amount for the policy is 3 and the probability of a claim for a policy is 0.1.
(a) Suppose X is the company's risk exposure for a given individual. Find $E[X]$, $\text{var}[X]$ and $E[(X - E[X])^3]$.
(b) The company decides to approximate the aggregate claim of the 100 individuals by a translated gamma variable with parameters (r, λ). Find the value of λ.

(4) An individual automobile insured has a claim count distribution per policy period that follows a Poisson distribution with parameter Λ. For the overall population of automobile insureds, the parameter Λ follows a distribution with density function $f(\lambda)$,

$$f(\lambda) = 9\lambda e^{-3\lambda} , \quad \lambda > 0 .$$

One insured is selected at random from the population and is observed to have a total of 2 claims during 5 policy periods. Determine the expected number of claims that this same insured will have during the 6th policy period.

(5) An investor keeps his assets either in cash or stocks. The annual rate of interest on cash is 7 percent. For stocks assume that the annual rate of return is 15 percent with probability 2/3, and 3 percent with probability 1/3. The investor takes all annual gains as income. At the end of each year he tranfers his principal from cash to stocks with probability 3/4, or from stocks to cash with probability 2/5. Suppose he starts with 100,000 dollars in cash. Estimate how much income he derived from the investment during the first 3 years.

(6) The number of claims a person makes to a medical insurance company is a Poisson process with a frequency of 3 claims per year.
(a) Suppose the person made a claim on April 25th, 2000. What is the probability the person will make another claim within the next 100 days.
(b) Suppose the person has made no claims during the first 200 days of the year 2000. Find the probability that the person will make at most 1 claim during the year 2000.

(7) A surplus process has a compound Poisson claims process with claim variable uniformly distributed in the interval $[0, 1]$. Suppose the adjustment coefficient is 0.8.
(a) Find the security loading on the policy.
(b) If the premium collection rate is 5 per unit time, find the claim frequency rate.

(8) A compound Poisson claims process has exponential claim variable with mean $1/3$. The claim frequency is 6 and premium collection rate is 2.4.
(a) Find the adjustment coefficient for the process.
(b) Find how much initial reserve the company requires to make the probability of ruin less than 5 percent.