

Math 623, F 2013: Homework 2

For full credit, your solutions must be clearly presented and all code included.

- (1) Consider the situation described in problem (2) of homework I. In this problem we shall use the Euler method from there (modified appropriately), to find the value of an American put option. Suppose K is the strike price of the option.
 - (a) Write down the boundary and terminal data corresponding to (c) of problem 2 in homework I, which one needs to find the value of the American option.
 - (b) Write down the explicit finite difference algorithm (modified from the one in (d) of problem 2 in homework I) which you will use to numerically compute the value of the option.
 - (c) Now take $K = 20$, $\sigma = 0.32$, $T = 0.5$ in (b). Implement the scheme with $\Delta x = (b - a)/2^5$ and for the value $\alpha = 8$. For a given value of r , plot the graphs of the computed value of the American option, and the value (using the MATLAB function *blsprice* for example) of the corresponding European put option against stock price on the same axis. Compare the two graphs for the three values of r , $r = 0$, $r = 0.05$, $r = 0.1$, and give an explanation.
 - (d) For $r = 0.05$, estimate by how many percent the value of the American option exceeds the value of the corresponding European option for the following values of today's stock price: $S = 15, 20, 25$, and give an explanation. Use $\Delta x = (b - a)/2^9$ here.
 - (e) For $r = 0.05$ plot the graph of the boundary of the no-exercise region in the (S, t) -plane, $0 < t < T$, i.e. the region where it is *not* optimal to exercise the option. Again use $\Delta x = (b - a)/2^9$ here.

- (2) This problem deals with an *continuously sampled, arithmetically averaged, average strike Asian call option*. Today is $t = 0$ and the option expires at time T years. The rate of interest is r and the volatility of the stock is σ . Dividends on the stock are paid out at a constant rate D .

Write $I_t = \int_0^t S_u du$, where S_t is the price of the stock at time t . Denote the option price at time by $V(S, I, t) = V(S_t, I_t, t)$. The PDE for V is given by

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + (r - D)S \frac{\partial V}{\partial S} + S \frac{\partial V}{\partial I} - rV = 0.$$

- (a) What is the terminal condition of the PDE, i.e. what is $V(S, I, T)$?
- (b) Exploit the homogeneity in the problem and use a similarity reduction as follows. Write $\xi = I/S$ and define the function $W(\xi, t)$ by $V(S, I, t) = SW(I/S, t)$. Derive the PDE which W must satisfy.
- (c) What is the terminal condition for the PDE in (b)?
- (d) Truncate the domain of W to $0 \leq t \leq T$ and $0 \leq \xi \leq \xi_{\max}$. Why could $\xi_{\max} = T(1 + 3\sigma\sqrt{T})$ be a reasonable value and what boundary condition for W at $\xi = \xi_{\max}$ could we use?
- (e) Write down the implicit boundary condition at $\xi = 0$ resulting from the PDE with $\xi = 0$, there.
- (f) Write down (carefully) an explicit finite difference scheme for the PDE in (b)-(e) in which the first derivative is approximated by the forward difference. What relationship between Δt and $\Delta \xi$ does the “rule of thumb” indicate that we should use? Show that the difference scheme is stable.
- (g) Implement the finite difference scheme in (f) for $T = 0.8$, $\sigma = 0.35$, $r = 0.052$, $D = 0.02$, choosing $\Delta \xi = \xi_{\max}/2^9$, and find the corresponding value of the option when the stock price today is 22.
- (h) Write down (carefully) an explicit finite difference scheme for the PDE in (b)-(e) in which the first derivative is approximated by the central difference for $\xi > \sqrt{\Delta \xi}/\sigma$ and by the forward difference when ξ is smaller, and show that this scheme is also stable. Implement the scheme for the same values of the parameters as in (g) and find the value of the option.
- (i) Using either the scheme in (f) or (h) plot the graph of the value of the option against volatility, with σ varying in the range $0 < \sigma < 1$ and the other variables maintaining their given values.