Math 623, F 2013: Homework 5.

For full credit, your solutions must be clearly presented and all code included.

(1) This problem deals with the pricing of a strangle option using a binomial tree. The underlying stock price S_t follows geometric Brownian motion with volatility $\sigma = 0.28$ (in units year$^{-1/2}$) and the interest rate is $r = 5.24\%$ per year, continuously compounded. The stock pays a continuous dividend yield of $D = 1\%$ per year. It is currently trading at $S_0 = 36$.

The option pays $\Phi(S)$ if exercised when the stock price is S. Here

$$\Phi(S) = \begin{cases} 30 - S & \text{if } 0 \leq S \leq 30 \\ 0 & \text{if } 30 < S \leq 40 \\ S - 40 & \text{if } S > 40. \end{cases}$$

Today is $t = 0$. The option expires in $T = 1/2$ years.

(a) Using the Black-Scholes formulas, find the exact value of the *European* strangle option today.

(b) Construct a binomial tree with time step $\Delta t = T/2^{10}$. Compute the parameters p_u, p_d, u, d and the corresponding value of the option under the following conditions:

(i) The (noncentral) moments of $S_{t+\Delta t}/S_t$ of order 0, 1 and 2 in the tree equals the corresponding moments under geometric Brownian motion. Moreover, $ud = 1$.

(ii) The (noncentral) moments of $S_{t+\Delta t}/S_t$ of order 0, 1 and 2 in the tree equals the corresponding moments under geometric Brownian motion. Moreover, $p_u = p_d = 1/2$.

(c) Implement the binomial trees constructed in (b) to value the corresponding American strangle option today.

(2) The following table gives the yields of zero coupon bonds, computed using *annual* compounding, for maturities up to 20 years. The table also gives the market Black implied (spot) volatilies of at-the-money caplets, each with 1 year accrual period.

Maturity	Yield	Volatility	Maturity	Yield	Volatility
1	0.0532	—	11	0.0602	0.1510
2	0.0529	0.1508	12	0.0594	0.1500
3	0.0534	0.1530	13	0.0585	0.1480
4	0.0546	0.1580	14	0.0575	0.1450
5	0.0562	0.1630	15	0.0569	0.1410
6	0.0579	0.1605	16	0.0561	0.1365
7	0.0596	0.1585	17	0.0551	0.1315
8	0.0606	0.1552	18	0.0546	0.1255
9	0.0609	0.1530	19	0.0543	0.1220
10	0.0606	0.1520	20	0.0543	0.1180

(a) Use the table to compute the swap rate on a 17 year loan, with interest payable annually.

(b) Compute the swap rate for a swap beginning in 5 years and lasting 8 years, with interest payable annually.

(c) Suppose the Black volatility for a 5×8 European swap option is 0.183. Find the cost (on a notional principle of 100) of a prepayment option after 5 years (swaption) on a 13 year fixed rate loan where the rate is 5% payable annually.

(d) Using the caplet volatilities in the table, find the cost (on a notional principle of 100) of an interest rate cap on a 17 year floating rate loan where the cap is 6% annual rate.

(3) In this problem we shall construct a Hull-White tree with the parameters $\sigma = 0.01$ and $a = 0.1$. The tree will be calibrated for $0 \le t \le T$ with $T = 18$, using the yield curve of problem 2. Yields for maturity dates which are not given are to be obtained by interpolation, with the yield for the shortest term bond assumed to be the same as the yield on the one year bond (0.0532).

(a) For $\Delta t = 1/2^n,\ n = 2, \ldots 8$, calculate the corresponding values of Δr and the two values of J, namely the Hull-White value $J = [1 - \sqrt{2/3}\ ]/(a\Delta t)$ and the generally smaller value $J = \sqrt{3/(2a\Delta t)}$.

(b) Taking J to be as in the Hull-White implementation of the model, calculate the values of the functions $\alpha_n(t)$ with $t = 12$ for the various values of $\Delta t = 1/2^n$ given in (a). You should do this in the two cases where intermediate yields are obtained using (1) linear interpolation, (2) spline interpolation.

(c) Plot on the same axes the graphs of the functions $\alpha_n(t),\ 0 < t < 12$, for $n = 8$ corresponding to the two types of interpolation in (b).

(d) Plot on the same axes the graphs of the functions $\alpha_n(t),\ 0 < t < 12$, for $n = 8$ corresponding to the two J values in (a) and spline interpolation used for intermediate yield values.