

Math 623, F 2013: Homework 6.

For full credit, your solutions must be clearly presented and all code included.

- (1) This problem is concerned with calculations on the Hull-White tree constructed in problem 3 of homework V. You should use spline interpolation and the Hull-White value for J .
 - (a) Using the Monte-Carlo method, estimate the probability $p_n(T)$ that the interest rate $r(t) < 0$ for some $t \leq T$. Do this for $n = 2, \dots, 6$, and $T = 12$. Use $N = 10^5$ simulations in the MC implementation.
 - (b) Plot on the same axes the graphs of $p_n(T)$, $0 \leq T \leq 12$ for $n = 2, 3, 4$.
 - (c) Using the Monte-Carlo method, estimate the value of an interest rate cap (on a notional principle of 100) for a 12 year floating rate loan where the cap is 6% annual rate, with payment of interest 8 times per annum. Use $N = 10^5$ simulations in the MC implementation.
 - (d) Estimate the value of the cap in (c) using the finite difference method.
- (2) This problem is also concerned with calculations on the Hull-White tree constructed in problem 3 of homework V. You should use the finite difference method (as opposed to Monte-Carlo) to solve the problem. In parts (a) and (b) you should use spline interpolation and the Hull-White value for J . Give your answers for $n = 2, \dots, 8$.
 - (a) Compute the value of a bond call option. The option may be exercised 6 years from today (and only then). When exercised, the holder of the option may buy for 60 a zero coupon bond, with a face value of 100 maturing at time $T = 14$ years.
 - (b) Find the cost (on a notional principle of 100) of a prepayment option (swaption) after 5 years on a 13 year fixed rate loan where the rate is 5% payable annually.
 - (c) Repeat parts (a) and (b) using the alternative value $J = \sqrt{3/(2a\Delta t)}$ in the Hull-White model.
- (3) This problem is concerned with the BDT model. The BDT tree will be calibrated using the yield and volatility curves of problem 2 of homework 5. Yields for maturity dates and volatilities for time intervals not given are to be obtained using spline interpolation. The yield on the shortest term bond is assumed to be the same as the yield on the one year bond (0.0532). Similarly, the Black implied volatility for the first time interval is assumed to be the same as the implied volatility for year 1-2 i.e. 0.1508.
 - (a) With $\Delta t = 1/16$ and $T = 14$, plot on the same axes the graphs of the forward rates for the shortest time intervals and the mean of the BDT short rate against time. Give numerical values for the forward rate and BDT short rate mean at time $t = 10$ years.

- (b) With T and Δt as in (a) plot the graph of the ratio of the forward rate to the BDT short rate mean against time.
- (c) Find the cost (on a notional principle of 100) of an interest rate cap on a 17 year floating rate loan where the cap is 6% annual rate. Give your answers for $\Delta t = 1/2^n$ with $n = 1, 2, 3, 4, 5$. Compare your answer to the answer you obtained for homework 5 problem 2 (d) and give an explanation.
- (d) Find the cost (on a notional principle of 100) of a prepayment option (swaption) after 5 years on a 13 year fixed rate loan where the rate is 5% payable annually. Give your answers for $\Delta t = 1/2^n$ with $n = 1, 2, 3, 4, 5$.