# New Algorithms in Diffusion Generated Motion

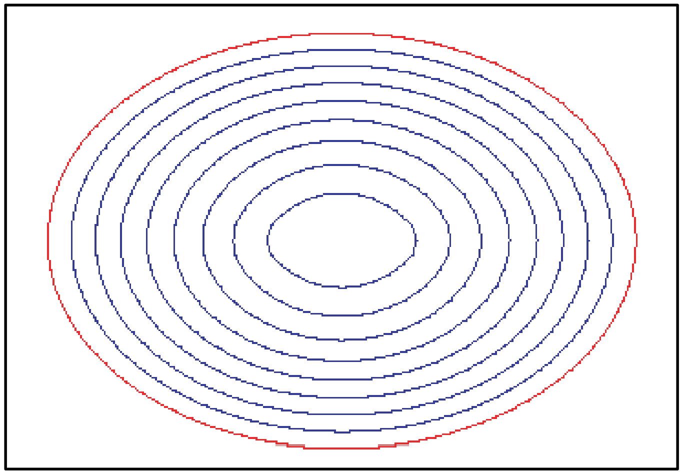
Selim Esedoglu (DMS-0713767), Steven Ruuth, and Richard Tsai

We have developed a new, more accurate version of Merriman, Bence, and Osher's threshold dynamics algorithm for evolving interfaces under geometric motions. The new version also alternates two computationally efficient operations -- construction of the signed distance function, and convolution with a kernel -- to generate a variety of geometric motions accurately on uniform grids.

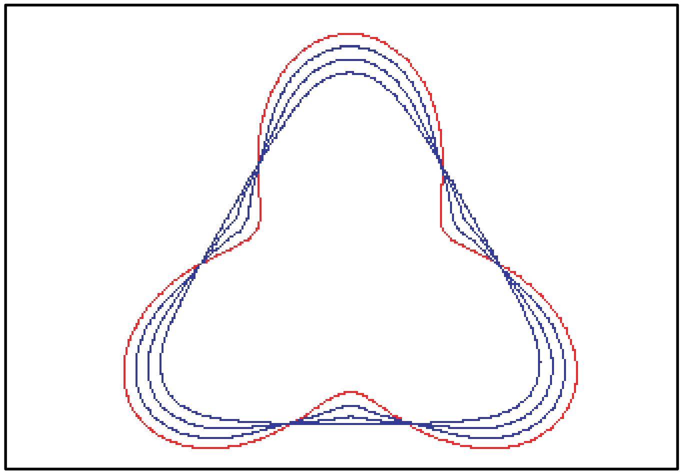
For example, the *affine invariant curvature motion* of a planar curve $\Gamma_0$ moves the curve with normal speed $\kappa^{1/3}$ where $\kappa$ is its curvature. This motion is of fundamental importance in many computer vision tasks. With our approach, it can be approximated as follows: Given the signed distance function $d_{n-1}$ to the solution $\Gamma_{n-1}$ at time $(n-1)(\delta t)$, construct the signed distance function $d_n$ to the solution $\Gamma_n$ at the next time step by the following prescription:

1. Form the convolution $u(x)=G_{\delta t} * d_{n-1}$
2. Let $d_n(x)$ be the signed distance function to
   $$\left\{x : d_{n-1}(x) + \left(u(x)-d_{n-1}(x)\right)^{1/3} (\delta t)^{2/3} = 0\right\}.$$

where $G_t(x)=(4\pi t)^{-1} \exp(-|x|^2/4t)$ is the Gaussian kernel in 2D.



Evolution of an initial ellipse (red). Thanks to affine invariance of the flow, it stays an ellipse of constant eccentricity as it shrinks, (blue curves) unlike in regular curvature flow.



Evolution of a more interesting initial curve (red) under the affine invariant flow.