Finding Global Minimizers of Segmentation Models

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Many variational models for image segmentation are non-convex and are known to possess local minima. These models are often solved by iterative energy minimization; successful results therefore require a good initial guess. An example is the two-phase piecewise constant Mumford-Shah model that aims to separate a given image f(x) into two regions: Foreground and background.

Motivated by previous work of T. Chan and L. Vese, we developed an algorithm that helps avoid local minima of this model. When the average intensities c_1 and c_2 of the foreground and background in image f(x) are known, it is guaranteed to find a global minimizer. The idea is to minimize the following convex energy:

$$u(x) = \underset{0 \le v(x) \le 1}{\operatorname{argmin}} \int |\nabla v| + \lambda \int \left[(c_1 - f)^2 - (c_2 - f)^2 \right] v \, dx$$

Using observations made in previous works of G. Strang, we proved that cross-sections of u(x) depict the boundary between foreground and background regions of a global minimizer of the original model. Our technique thus significantly reduces the dependence of results on initial guess.

Original Image f(x)

A Cross-Section of u(x)