Higher Order Image Decomposition Models and Their Solution via Convex Duality

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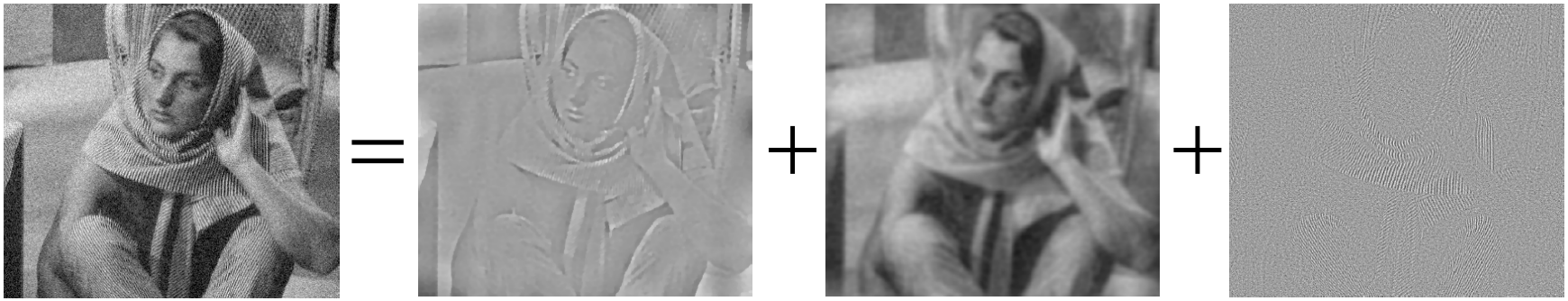
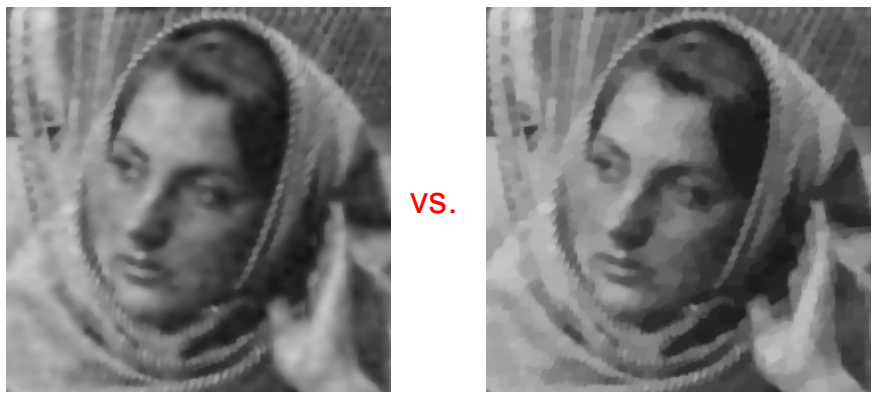


Noisy Image f(x) $u_1(x)$ $u_2(x)$ Texture + Noise

Variational image denoising models based on the total variation model of Rudin, Osher, and Fatemi have seen much recent progress in how they handle small scale image features. The work of Osher, Sole, and Vese (OSV) and others concentrated on improving separation of textures from the piecewise smooth component of a given image by the use of negative Sobolev norms. Another important caveat of these models, unaddressed by recent research on negative norms, is *staircasing*: Smooth regions of denoised images take on a patchy appearance. Building up on previous ideas of Chambolle and Lions, we show how staircasing can be prevented at the same time by the inclusion of a higher order term into the model that is designed specifically to capture the smooth part of the given image. Our model consists in minimizing:

$$\inf_{u_1,u_2} \left\{ \int |\nabla u_1| + \alpha \int |\Delta u_2| + \frac{1}{2\lambda} \int \left| \nabla \Delta^{-1} (f - u_1 - u_2) \right|^2 dx \right\}$$

Both the u_1 and the higher order u_2 term in the energy are non-smooth. Following Chambolle and others, we developed an algorithm based on *convex duality* to cope with this difficulty.



Our $u_1(x)+u_2(x)$ vs. Denoised by OSV