## Exam 1 Review Problems

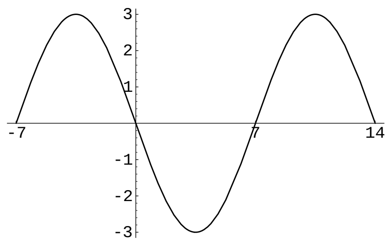
(this list is guaranteed to be incomplete, with problems that may or may not reflect the actual level of difficulty of the exam.)

1. For each of the following tables of data, determine what type of function is most likely to have generated it. Explain your answer. Then find a formula for the function.

| | | | |
|---|---|---|---|
| $x =$ | 1.2 | 1.6 | 2.2 |
| $f(x) =$ | 4.88 | 5.74 | 7.32 |

| | | | |
|---|---|---|---|
| $x =$ | $-2$ | $-1.6$ | $-1$ |
| $g(x) =$ | 4.8 | 4.28 | 3.5 |

2. Find *two* possible functions that could give the graph below. Is this function invertible? Why or why not?




3. Sketch a graph of the function $f(x) = \begin{cases} x+2 & \text{if } -3 \le x < 0 \\ 2 & \text{if } 0 \le x < 1 \\ 2-(x-1)^2 & \text{if } 1 < x < 3 \end{cases}$. Then sketch graphs of each of (a) $f(x)+2$, (b) $f(x+2)$, and $\frac{1}{2}f(\frac{x}{3})$.

4. If you restricted the domain for the function $f(x)$ in problem (3), could you come up with a new function that is invertible? If so, what new domain works? If not, why not?
   a. Sketch the inverse function.

5. Solve for $x$ in each of the following. (a) $2\,(3^x) = e\,(2^x)$; (b) $5 = 4\,e^{1.5x} - 3$; (c) $3x + 2 = 4e^{2\ln(x)}$.

6. Find the inverse function for the function $\text{invertme}(x) = 3(6^{x/2})$.

7. Suppose that the number of students getting "A"s in math 115 is increasing at a continuous rate of 3% per year[1]. How long would it take for the number to increase 50%? 100%? If the initial number was 5% of the students taking the class, when would it reach 10%? What would we call this time?

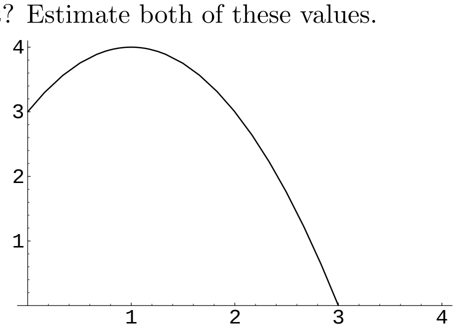
8. Without your calculator, sketch the function $t(x) = 105 + 800\cos(3x-6)$. What are the amplitude and period of $t(x)$?

9. Does it make sense to talk about the amplitude or period of the tangent function $\tan(x)$? Explain.

10. Solve $3 = 9\sin(2x-1) + 4$ for $x$. Give your answer in exact and in decimal form.

11. For problem (1), suppose that the first graph had been generated by a function $h(x) = ax^2 + bx + c$. What are $a$, $b$ and $c$? Explain why or why not you think this is a better answer than that which you gave in problem (1) (assuming, of course, that it is different).

12. Consider the function $s(t)$ graphed below. Suppose that this is the vertical position (in meters) of an Purple and Chartreuse Indonesian Sparrow as a function of time $t$ (in minutes). When is its vertical velocity the largest? Smallest? Estimate both of these values.




[1] *This is not in any way guaranteed to actually be the case.*

a. (Continued from problem (12).) Estimate the average velocity of the sparrow for each of the time intervals $0 \le t \le 1$, $1 \le t \le 2$, and $1 \le t \le 2$. Illustrate on a copy of the figure how your answers relate to the graph.

b. For what times, if any, is the *speed* of the sparrow greater than one?

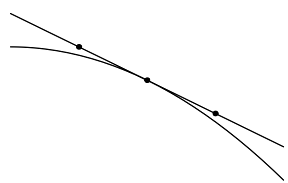
13. Estimate to two decimal places $d'(2)$ if $d(x) = 3\left(\frac{1}{2}\right)^x$.

14. Estimate $q'(3)$ if $q(x)$ is given by

| | | | | | |
|---|---|---|---|---|---|
| $x =$ | 2.9 | 2.99 | 3 | 3.01 | 3.1 |
| $q(x) =$ | 7.22 | 7.9202 | 8 | 8.0802 | 8.82 |

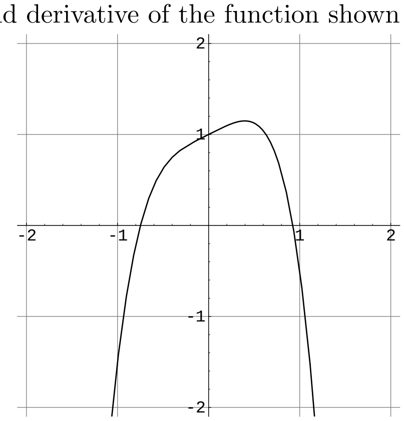
a. Find the equation to the tangent line to $q(x)$ at $x = 3$.

15. Suppose in the graph below the line shown is tangent to the graphed function $g(x)$. If the three points are, respectively, $(0.9, 1.6)$, $(1, 1)$, and $(1.1, 0.4)$, what are $g(1)$ and $g'(1)$? What can you say about $g''(1)$?



16. Suppose that the function graphed in problem (2) is $\text{fred}(x)$. Sketch $\text{fred}'(x)$. Then sketch $\text{fred}''(x)$.

17. Sketch the derivative and second derivative of the function shown below.




a. Where is this function concave up? Concave down?

18. Do a bunch of problems from §2.5.

19. Now suppose that the graph in (17) is a graph of $f'(x)$ for some function. Sketch $f''(x)$.

a. When is $f(x)$ largest on the range $-1 \le x \le 1$? Smallest?

b. Identify where $f(x)$ is concave up and concave down.

20. Now suppose that the graph in (17) is the *velocity* function of a Plaid Orange and Lime Coated Math Professor. Are there any points

a. where the Professor's velocity is zero?

b. where the Professor's acceleration is zero?

21. Suppose that $E(c)$ is the ecstacy, measured in deliriums, of a calculus student who has studied $c$ chapters of calculus. What are the units of $E'(c)$ and $\frac{d^2E}{dc^2}$? What is the practical meaning of $E(2) = 1.78$? Of $E'(2) = 0.56$?

a. What are the units of $E^{-1}(2)$? What are the units and the meaning of $E^{-1}(2) = 2.5$? Give a practical interpretation of the statement $(E^{-1})'(2) = 0.8$.