MATH 115-027 QUIZ 2 / 27 Sep 2005 Name:________________________________

1. (4 points) Suppose that the number of students at the University is a growing exponential function with a doubling time of 85 years. If there are currently 24,000 students, how long will it take for there to be 30,000?

Solution: We know that the population doubles every 85 years, so the easiest exponential function is $P(t) = 24,000 \cdot 2^{t/85}$, where t gives the number of years from the present. We could also write a natural exponential: if $P(t) = 24,000e^{kt}$, then $P(85) = 48,000 = 24,000e^{85k}$, so $e^{85k} = 2$, or $k = \frac{1}{85}\ln(2)$. In either case, we want to know when $P(t) = 30,000$. With the first equation, $24,000 \cdot 2^{t/85} = 30,000$, or $2^{t/85} = \frac{5}{4}$, so $\frac{t}{85}\ln(2) = \ln(\frac{5}{4})$, so $t = 85\frac{\ln(5/4)}{\ln(2)} \approx 27.36$ years. (Note that with the natural exponential we get the same thing: $P(t) = 24,000e^{kt} = 30,000$, so $kt = \ln(\frac{5}{4})$, but $k = \frac{1}{85}\ln(2)$, so this is exactly the same equation as we obtained before.)

---

2. (4 points) A student in a psychology course observes that students' average grades on daily quizzes vary sinusoidally with the day of the week on which the quizzes are taken. If the minimum average grade is 65% and is obtained on Friday while the maximum is 80%, obtained on Wednesday, write a formula for the average grade, A as a function of of the day of the week t. (Note that in this case it is reasonable to consider a five-day week.)

Solution: The midline of the oscillation is $\frac{1}{2}(65+80) = 72.5\%$, and the amplitude is $\frac{1}{2}(80-65) = 7.5\%$. Let $t = 0$ be Monday. Then $t = 2$ is Wednesday, and $t = 4$ is Friday. We know that the minimum occurs on Friday, so we might choose to write the oscillation as a negative cosine function. The minimum occurs when $t = 4$, so we shift it four units to the right: $A(t) = 72.5 - 7.5\cos(B(t-4))$. If the grades repeat every week, the period of the oscillation is 5 days, so $5B = 2\pi$, or $B = \frac{2\pi}{5}$, and we are left with $A(t) = 72.5 - 7.5\cos(\frac{2\pi}{5}(t-4))$. (We note that Wednesday is $t = 2$, where $A(2) \approx 78.6\%$, which is less than the maximum we know is supposed to occur there. This isn't a problem, however; we're modeling "real" data, so that we don't necessarily expect exact agreement.)

---

3. (2 points) Estimate $\lim\limits_{x\to 0} \frac{\ln(x+1)}{2x}$ to within 0.01. If the limit does not exist, explain how you know.

Solution: We can estimate the limit by plugging in successively smaller values of x. With $x = 0.01$, we have $\frac{\ln(1.01)}{0.02} \approx 0.4975$, or, to 0.01, 0.50. With $x = 0.001$, we have $\frac{\ln(1.001)}{0.002} \approx 0.5000$. These agree to 0.01, so we are happy to assert that $\lim\limits_{x\to 0} \frac{\ln(x+1)}{2x} = 0.50$ (to within 0.01).