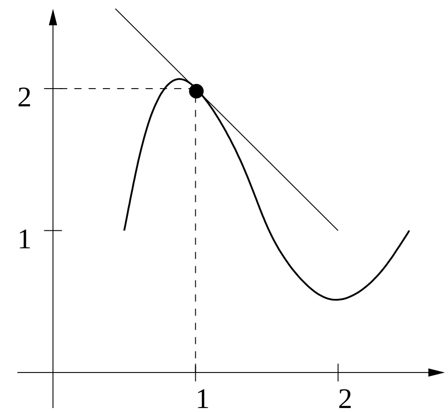
MATH 115-027 QUIZ 3 / 6 Oct 2005 Name:______________________________

For the first two problems, consider the graph to the right.



1. (2 points) Suppose that the graph shows a function $f(x)$ and a line tangent to the curve $y = f(x)$ (the solid line). If the equation of the line is $y = 3 - x$, fill in the following blanks:

$f(\_\_\_) = \_\_\_$
$f'(\_\_\_) = \_\_\_$

Solution: The tangent line goes through the point $(1, 2)$ on the graph, so $f(1) = 2$. The slope of the tangent line is $m = -1$, which must be the same as the slope of $f(x)$ at $x = 1$, so $f'(1) = -1$.

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2. (4 points) Now suppose that the curve graphed is $g'(x)$ for some function g. The indicated line is still $y = 3 - x$, and is tangent to the graph of $y = g'(x)$. Assume that $g(x)$ and its derivatives are defined only for $\frac{1}{2} \le x \le 2\frac{1}{2}$. Where (for what x-value) is

$g(x)$ smallest?	$g'(x)$ smallest?	$g''(x)$ smallest?
$g(x)$ largest?	$g'(x)$ largest?	$g''(x)$ largest?

Are the largest values of $g'(x)$ and $g''(x)$ greater than 1? Greater than 2? Greater than 3? (How do you know?)

Solution: Because this is a graph of $g'(x)$, and because the graph is always above the x-axis, we know that $g(x)$ must always be increasing. Therefore the smallest value of $g(x)$ occurs at $x = \frac{1}{2}$ and the largest at $x = 2\frac{1}{2}$. The smallest and largest values of $g'(x)$ may be read directly from the graph: the smallest value is at $x = 2$ and the largest is slightly before $x = 1$. The smallest and largest values of $g''(x)$ are where the slope of $g'(x)$ is the smallest and largest. It appears to be the smallest at a little before $x = 1.5$ and largest at about $x = \frac{1}{2}$.

The largest value of $g'(x)$ is larger than 2 (because the graph has a maximum above 2) but not larger than 3. The slope of the graph at $x = \frac{1}{2}$ appears to be well in excess of 3, so the largest value of $g''(x)$ is greater than 3.

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3. (4 points) A Purple-Headed Uniquely Nocturnal Chartreuse And Luridly Colored wombat is sighted moving across the diag. Its position, measured in feet from the West Engineering arch, is given as a function of time (in minutes past midnight) in the following table.

t	0	5	10	15	20	25	30
position	0	7	15	27	30	31	218

a. Estimate the wombat's velocity at $t = 0$, $t = 5$, $t = 10$ and $t = 15$.
b. Estimate the wombat's acceleration at $t = 5$ and $t = 10$.
c. What do you think happened between $t = 25$ and $t = 30$?

Solution: We estimate the velocity by finding the average velocity that we think will be closest to the wombat's instantaneous velocity. Thus we have

$v(0) \approx \frac{7-0}{5-0} = \frac{7}{5}$ ft/s, $\quad v(5) \approx \frac{15-0}{10-0} = \frac{3}{2}$ ft/s,

$v(10) \approx \frac{27-7}{15-5} = 2$ ft/s, and $\quad v(15) \approx \frac{30-15}{20-10} = \frac{3}{2}$ ft/s.

Similarly, we can approximate the acceleration by using the average accelerations:

$a(5) \approx \frac{2-\frac{7}{5}}{10-0} = \frac{3}{50}$ ft/s^2, and $\quad a(10) \approx \frac{\frac{3}{2}-\frac{3}{2}}{15-5} = 0$ ft/s^2.

I suspect that the wombat hitched a ride on a passing bicycle between $t = 25$ and $t = 30$, thus dramatically increasing its velocity.