MATH 115-027 QUIZ 4 / 20 Oct 2005 Name:________________________

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1. Suppose that the value of a highly-prized silver-plated author-signed calculus textbook is given, in dollars, by $V(t) = 100(1.05^t - 0.02t)$, where t is the number of years from the publication date of the text. At what rate is the value changing seven years after the book's publication? (3 points)

Solution: The rate of change is given by the derivative, which, using shortcuts, is $V'(t) = 100(\ln(1.05) \cdot 1.05^t - 0.02)$. Thus after five years, we have $V'(7) = 100(\ln(1.05) \cdot 1.05^7 - 0.02) \approx 4.87$ dollars/year. Thus after seven years, the book is appreciating at a rate of \$4.87/year.

2. For what values of x is $f(x) = 4x^2 - 3^x$ both increasing and concave up? (Use your knowledge of derivatives to answer this question—though your calculator may be useful as you work out your answer.) (4 points)

Solution: We know that the function is increasing when its derivative is greater than zero. Here, $f'(x) = 8x - \ln(3) \cdot 3^x$, so we want $8x - \ln(3) \cdot 3^x > 0$. Similarly, the function is concave up when its second derivative is greater than zero. Taking the derivative of $f'(x)$, we have $f''(x) = 8 - (\ln(3))^2 \cdot 3^x$, so we want $8 - (\ln(3))^2 \cdot 3^x > 0$, or $3^x < 8/(\ln(3))^2$. Taking the natural log of both sides and solving for x, this becomes $x < \ln(8/(\ln(3))^3)$, or, finding a decimal approximation, $x < 1.797$.

Using the derivative condition is, unfortunately, more difficult: it's not possible to explictly solve $8x - \ln(3) \cdot 3^x = 0$ for x. So let's approximate it on a calculator: graphing $y = 8x - \ln(3) \cdot 3^x$, we see that $y > 0$ for, approximately, $0.165 < x < 2.717$. From the condition on $f''(x)$, we know that $x < 1.797$, so the range of values on which $f(x)$ is both increasing and concave up is, approximately, $0.165 < x < 1.797$.

3. Given the following data for f, g, f' and g',
a. if $h(x) = f(x) \cdot g(x)$, find $h'(1)$.
b. if $p(x) = f(x)/g(x)$, find $p'(2)$.
(3 points)

x	1	2
f	6	4
f'	-2	-1
g	-4	-3
g'	3	5

Solution: We know from the product and quotient rules that

$$h'(1) = f'(1) \cdot g(1) + f(1) \cdot g'(1) = (-2)(-4) + (6)(3) = 8 + 18 = 26$$

and

$$p'(2) = \frac{f'(2) \cdot g(2) - f(2) \cdot g'(2)}{g(2)^2} = \frac{(-1)(-3) - (4)(5)}{9} = -\frac{17}{9}.$$