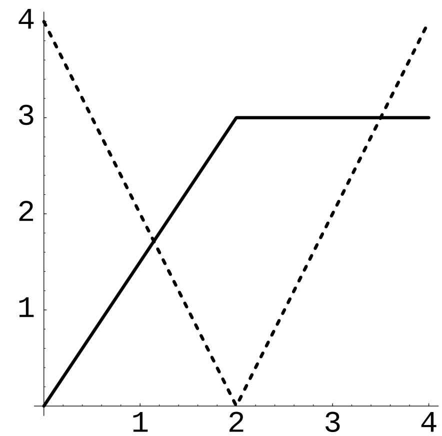
MATH 115-027 QUIZ 5 / 27 Oct 2005 Name:________________________

1. Consider the graphs of $f(x)$ (the solid curve) and $g(x)$ (the dashed curve) in the figure to the right. If $h(x) = g(f(x))$ and $v(x) = g(g(x))$, find (3 points)

a. $h'(1)$

b. $v'(3)$



Solution: For part (a), we know $h'(x) = g'(f(x)) \cdot f'(x)$, so $h'(1) = g'(f(1)) \cdot f'(1)$. Reading from the graph, we see that $f(1) = 1.5$, $g'(1.5) = -2$ and $f'(1) = 1.5$, so $h'(1) = (-2) \cdot (1.5) = -3$. For part (b), $v'(x) = g'(g(x)) \cdot g'(x)$, so $v'(3) = g'(g(3)) \cdot g'(3)$. Again, from the graph we have $g(3) = 2$, $g'(2)$ is undefined, and $g'(3) = 2$. Thus we get $g'(3)$ equal to the product of an undefined value and 2, which is undefined.

2. Suppose that the function $y = 3^{(-ax^2)}$ (with $a > 0$) is concave down for $-1 < x < 1$. What is a? You should use your penetrating knowledge of derivative short-cuts in the course of solving this problem. (4 points)

Solution: We know that the function is concave down when the second derivative is negative. Here, $\frac{dy}{dx} = -2ax \cdot \ln(3) \cdot 3^{(-ax^2)}$, so $\frac{d^2y}{dx^2} = -2a\ln(3) \cdot 3^{(-ax^2)} + 4a^2x^2 \cdot (\ln(3))^2 3^{(-ax^2)}$. For this to be negative, we have

$$-2a\ln(3) \cdot 3^{(-ax^2)} + 4a^2x^2 \cdot (\ln(3))^2 3^{(-ax^2)} < 0, \quad \text{so}$$
$$4a^2x^2 \cdot (\ln(3))^2 3^{(-ax^2)} < 2a\ln(3) \cdot 3^{(-ax^2)}.$$

We know that 2, a, $\ln(3)$ and $3^{(-ax^2)}$ are all positive, so we can divide them out from both sides of the inequality to get

$$2ax^2 \cdot \ln(3) < 1, \quad \text{or} \quad x^2 < \frac{1}{2a\ln(3)}.$$

This gives $-1 < x < 1$ if $\frac{1}{2a\ln(3)} = 1$, or $a = \frac{1}{2\ln(3)}$.

3. Suppose that students' ecstasy about being in calculus, E, is given (in deleriums, the standard unit for ecstasy) by $E(t) = \frac{1}{1+\cos(\frac{\pi t}{3})}$, where t is the number of weeks since the start of the semester. At what rate is the students' ecstasy changing seven weeks into the term? Three weeks into the term? Explain why these two values make sense by referring to the function $E(t)$. (3 points)

Solution: We can find the rate of change of E by finding $E'(t)$. Taking the derivative,

$$E'(t) = \frac{\frac{\pi}{3}\sin(\frac{\pi t}{3})}{(1+\cos(\frac{\pi t}{3}))^2},$$

so $E'(7) \approx 0.4031$ deleriums/week, and $E'(3)$ is undefined! Considering $E(t)$ explains this: at $t = 3$, the denominator of $E(t)$ becomes zero, and there is a vertical asymptote. This corresponds to the undefined value of the slope at $t = 3$. At $t = 7$ we are close to $t = 6$, where $\sin(\pi t/3)$, and therefore $E'(t)$, is zero. The function $E(t)$ has a minimum at $t = 6$.