MATH 115-027 QUIZ 6 / 8 November 2005 Name:____________________

1. For some $k > 0$, the functions $f(x) = 2e^x$ and $g(x) = kx$ are tangent for a value of $x > 0$. Find the values of x and k that result in this condition being true. (4 points)

Solution: If $f(x)$ and $g(x)$ are tangent, we know that they intersect and that their slopes are the same at the point of intersection. Call the intersection point $x = a$ (which is the point of tangency). Then, at $x = a$, $f(a) = g(a)$ and (to match slopes) $f'(a) = g'(a)$. This requires that $2e^a = ka$ and $2e^a = k$. Dividing the first equation by the second, we get $a = 1$, so that, plugging back in to either equation, $k = 2e^1 = 2e$.

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2. Find the linear approximation to the function $h(x)$ defined implicitly by $x^2y + 3xy^4 = 10$ if we are interested in values of x and y near the point $(2, 1)$. (3 points)

Solution: The linear approximation is just the tangent line at $x = 2$. To find this, we need a slope and the point $(2, 1)$. The slope we can find by implicit differentiation. Differentiating both sides of the equation, we get $2xy + x^2\frac{dy}{dx} + 3y^4 + 12xy^3\frac{dy}{dx} = 0$, so that $\frac{dy}{dx} = -\frac{2xy+3y^4}{x^2+12xy^3}$. At $(x, y) = (2, 1)$, this is $\frac{dy}{dx} = -\frac{4+3}{4+24} = -\frac{7}{28} = -\frac{1}{4}$. Therefore, using point-slope form, the linear approximation (tangent line) is $y = 1 - \frac{1}{4}(x - 2) = \frac{3}{2} - \frac{1}{4}x$.

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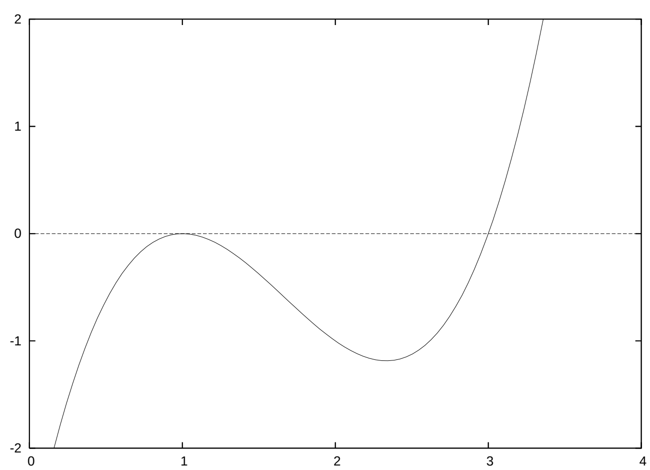
3. Suppose that the figure to the right shows $f'(x)$ for some function $f(x)$. Identify all critical points, local maxima and minima, and inflection points of the function $f(x)$. (3 points)



Solution: We know that critical points (where there may be local maxima or minima) occur when $f'(x) = 0$, so critical points are when $x = 1$ and $x = 3$. Then $f'(x)$ changes sign (from negative to positive) at $x = 3$, so $x = 3$ is a local minimum, while $x = 1$ (where $f'(x)$ does not change sign) is neither a local maximum or minimum. Inflection points of $f(x)$ occur when $f'(x)$ has a local maximum or minimum, so, from the figure, we know that there is an inflection point of $f(x)$ at $x = 1$ and at the local minimum of $f'(x)$, approximately $x = 2.25$.