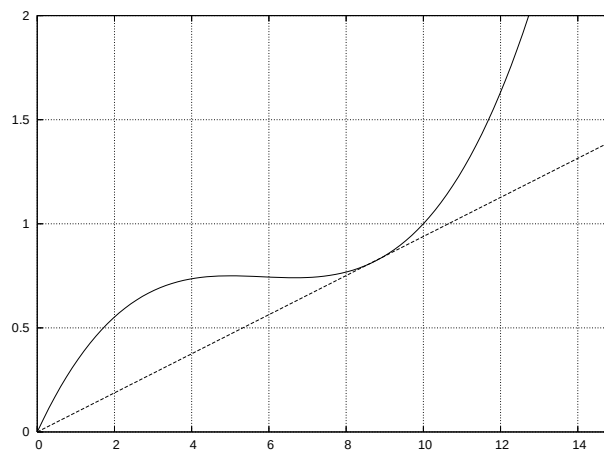


1. Consider the family of functions given by $f(x) = ax - e^{bx}$, where a and b are constants. What conditions on a and b guarantee that there will be at least one local maximum or minimum? In this case, how many local maxima are there, and where are they located? (4 points)

Solution: Given $f(x) = ax - e^{bx}$, $f'(x) = a - be^{bx}$. This has a zero if $a - be^{bx} = 0$, or, $e^{bx} = \frac{a}{b}$. Thus, because the exponential is always positive, and can take any positive value, there will be a critical point if $a \neq 0$ and a and b have the same sign. The critical point is in this case $x = \frac{1}{b} \ln(a/b)$. Then we can test it using the second derivative test: $f''(x) = -b^2 e^{bx} < 0$ for all values of x (and, therefore, for $x = \frac{1}{b} \ln(a/b)$), so this is a local maximum.

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2. Suppose that the graph given to the right shows $E(s)$, the total effort required to rouse s students from bed on a Thursday morning at some point in the semester. (The units of E are “Herculean tasks,” Ht.) How is the average effort required per student represented on this graph? What number of students requires the minimum average effort per student? (3 points)



Solution: The average effort per student will be $E(s)/s$, which is the slope of a line drawn from the origin to a point $(s, E(s))$ on the curve. The minimum such slope will occur as shown, when the line from the origin is tangent to the curve. This occurs at approximately $s = 8.5$ students. Which sounds a bit morbid. Let's say at $s = 9$ students, as the domain really ought to be in integer numbers of students.

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3. Suppose that the cost of producing q items of some product is given by $C(q) = 7 + 6q + 6q^2 + q^3$, where C is in thousands of dollars and q is in hundreds of the items produced. If each item is sold for 69 thousand dollars (no, really), write an equation for the profit obtained by selling the items. What number of items maximizes profit? (3 points)

Solution: We know that revenue is just the total amount that the q items are sold for, which in this case will be $R(q) = 69q$. Then the profit is revenue less cost, or $\pi(q) = 69q - (7 + 6q + 6q^2 + q^3) = -7 + 63q - 6q^2 - q^3$. The maximum profit will occur at critical points of this function (or at endpoints). The critical points occur when $\pi'(q) = 0$, or $63 - 12q - 3q^2 = 0$. Dividing by -3 , $q^2 + 4q - 21 = 0$, which factors as $(q - 3)(q + 7) = 0$, so $q = 3$ or $q = -7$. The latter doesn't lie in the domain (we can't produce a negative number of items), so we may ignore that. Then $\pi''(q) = -12 - 6q < 0$ for all values of q , so the profit function is always concave down and the critical point we found is a maximum. As it is the only critical point in the domain ($q \geq 0$), we know it must be the global maximum.