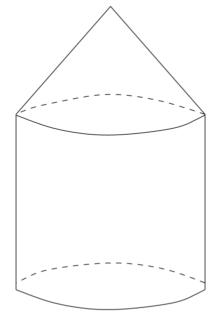
MATH 115-027 QUIZ 8 / 1 Dec 2005 Name:______________________________

1. Melvin the Moonshiner is constructing a new still for use in the backwoods. For the purposes of this problem, all you need to know about Melvin's still is that it is in the shape of a cylinder topped with a cone, as shown in the figure to the right. Melvin figures that the still must have a volume of 1 m^3, and in order to be able to carry it around, he must be able to reach around it, which requires it to be no more than 2 m around. Finally, to reduce the chance that it will be spotted, he wants the *cross-sectional area* of the still to be as small as possible. What dimensions should Melvin's still have? (It may be useful to note that the volume of a cylinder is $V_1 = \pi r^2 h$, while that of a cone is $V_2 = \frac{1}{3}\pi r^2 T$, where r is in either case the radius and h and T are the heights of the cylinder and cone, respectively.) (5 points)



Solution: Let the radius of the still be r, the height of the cylindrical portion be h, and the height of the cone be T. Then we know that the volume is 1 m^3 and the circumference is 2 m[†], so $1 = \pi r^2 h + \frac{1}{3}\pi r^2 T$ and $2 = 2\pi r$. Obviously, this says that $r = \frac{1}{\pi}$, so that, solving the first equation for h, we get

$$h = \frac{1 - \frac{1}{3}\pi r^2 T}{\pi r^2} = \pi - \frac{1}{3}T.$$

Note that $h \geq 0$, so $T \leq 3\pi$. We must also have $T \geq 0$, of course, so the end points for T are $T = 0$ and $T = 3\pi$.

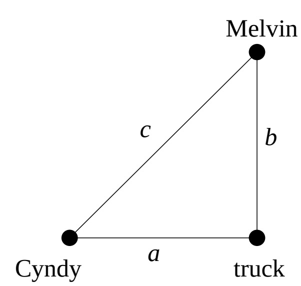
We want to minimize the cross-sectional area of the still, X, which is just the sum of the areas of a rectangle and a triangle, $X = 2rh + rT = \frac{2}{\pi}h + \frac{1}{\pi}T$. Substituting for h,

$$X = \frac{2}{\pi}(\pi - \frac{1}{3}T) + \frac{1}{\pi}T = 2 + \frac{1}{\pi}(1 - \frac{2}{3})T = 2 + \frac{1}{3\pi}T.$$

Thus $X'(T) = \frac{1}{3\pi}$, and there are no critical points. The minimum cross sectional area must occur at one of the end points. $X(0) = 2$ and $X(3\pi) = 5$, so the minimum is when $T = 0$ (there is no conical part to the still). The still is therefore a cylinder, with radius $r = \frac{1}{\pi}$ m and height $h = \pi$ m.

2. Cyndy the Surreptitious DEA Agent is investigating Melvin's still. Melvin suspects her presence when she is 500 feet southwest of the still. Melvin's pickup truck is due south of the still (and Melvin) and due east of Cyndy. If they at this moment both begin moving towards the truck, Cyndy at 5 ft/min and Melvin (trundling the still) at 3 ft/min, how fast is the distance between them changing? (5 points)

Solution: The situation is shown to the right. We can write an equation involving a, b and c, the lengths of the sides of the triangle, easily using the Pythagorean theorem: $a^2 + b^2 = c^2$. Differentiating both sides of this, we get $2a\frac{da}{dt} + 2b\frac{db}{dt} = 2c\frac{dc}{dt}$. We want to find $\frac{dc}{dt}$, the rate at which the distance between them is changing. When $c = 500$, $a = b = \frac{500}{\sqrt{2}}$ (because the triangle is, initially, isosceles), and we know that $\frac{da}{dt} = -5$ (negative because the distance is decreasing) and $\frac{db}{dt} = -3$. Thus, solving for $\frac{dc}{dt}$, we get $\frac{dc}{dt} = \frac{-8}{\sqrt{2}} \approx -5.7$ ft/min.



[†] *Really the problem only says that the radius is at most 2 m, but we'll assume that this should be an equality.*